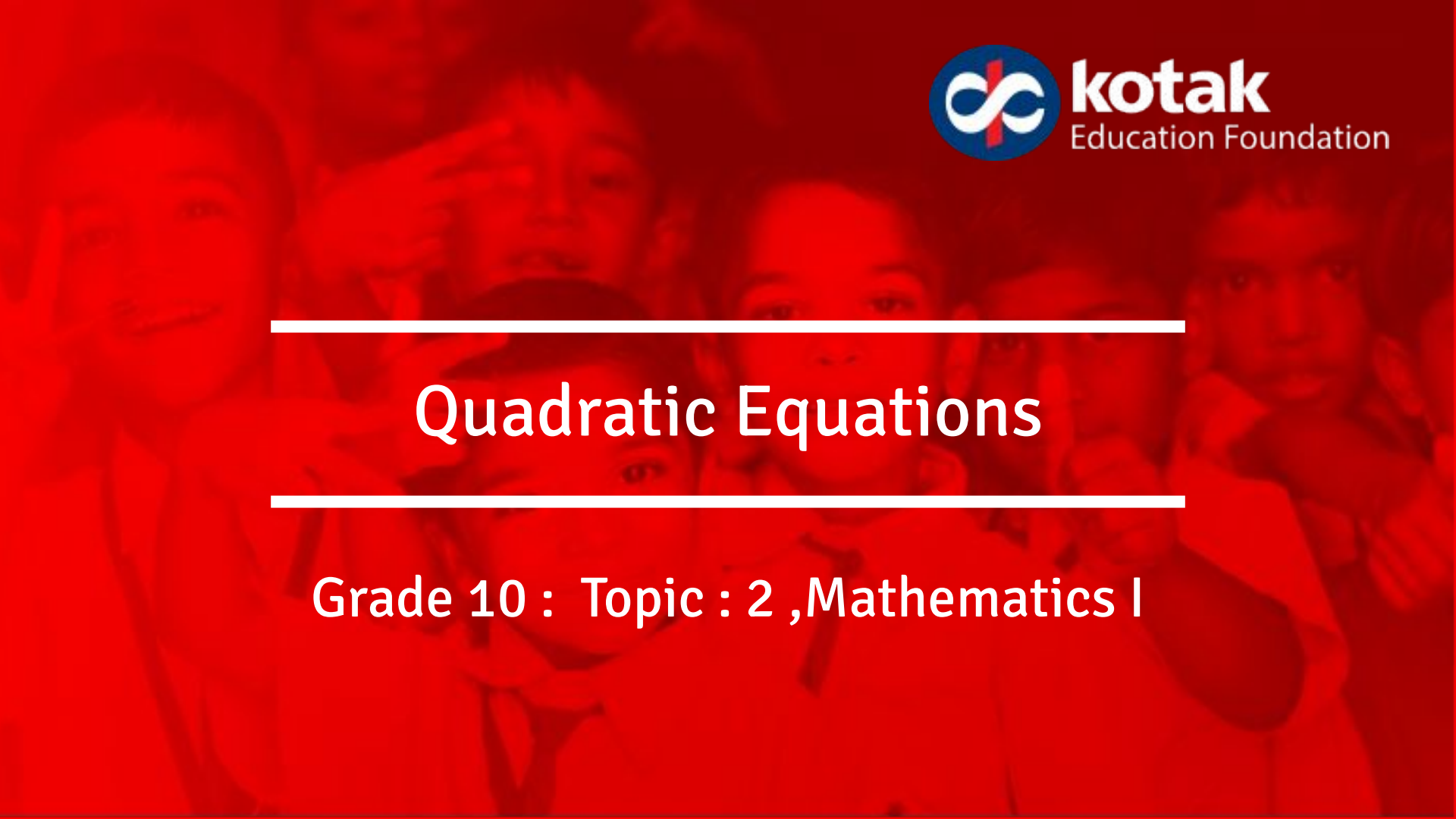

Quadratic Equations

Grade 10 : Topic : 2 ,Mathematics I

Topics : Quadratic equation

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1. Introduction
2. Methods of solving quadratic equation
3. Standard form of a quadratic equation
4. Solutions of quadratic equations by factorisation
5. Nature of roots of quadratic equation
6. Relation between roots and coefficients
7. Applications of quadratic equations

Standard form of a quadratic equation

- The name Quadratic comes from "quad" meaning square, because the variable gets squared (like x^2)
- It is also called an "Equation of Degree 2" (because of the "^{2}" on the x)

The Standard Form of a Quadratic Equation looks like this: Quadratic Equation:

$$ax^2 + bx + c = 0$$

a, b and c are known values. a can't be 0.

Standard form of a quadratic equation

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$$ax^2 + bx + c = 0$$

A General Quadratic Equation

Identify quadratic equations

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Equation	Is it Quadratic?	Explanation
$3x^3 - 4x + 5$	No	The first term is raised to the 3rd power. It must be raised to the 2nd power in order to be quadratic.
$5x^2 - 4x + 2$	Yes	This equation is in the correct form: $ax^2 + bx + c$
$7x^2 = 49$	Yes	This equation can be rewritten as: $7x^2 - 49$. In this equation, b is 0. B or c can be 0; however, a cannot be 0.
$2x^2 = 8x - 3$	Yes	This equation can be rewritten as $2x^2 - 8x + 3$ which would then be in the correct form of: $ax^2 + bx + c$.

Are these quadratic equations ?

- $X^2 + 5x - 14$
- $4y^2 - 7y - 11$
- $2y^2 - 12$

In your notebook write down if they are quadratic .

Also mention the reason

How to find a, b and c ?

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A	B	C	Quadratic Equation
1	-3	2	$x^2 + -3x + 2 = 0$
1	5	6	$x^2 + 5x + 6 = 0$
3	-6	3	$3x^2 + -6x + 3 = 0$
2	-4	7	$2x^2 + -4x + 7 = 0$
-3	-24	-48	$-3x^2 + -24x + -48 = 0$
1	2	3	$x^2 + 2x + 3 = 0$
1	2	0	$x^2 + 2x + 0 = 0$

Ways to solve a quadratic equations

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Solutions of a quadratic equation by factorisation

$$3X^2 - X - 10 = 0$$

$$\therefore \underline{3X^2 - 6X} + \underline{5X - 10} = 0$$

$$\therefore 3X(X - 2) + 5(X - 2) = 0$$

$$\therefore (3X + 5)(X - 2) = 0$$

$$\therefore (3X + 5) = 0 \text{ or } (X - 2) = 0$$

$$\therefore X = -\frac{5}{3} \text{ or } X = 2$$

$\therefore -\frac{5}{3}$, and 2 are the roots of the given quadratic equation.

Solve these equations

$x^2 - 15x + 54 = 0$

$x^2 + x - 20 = 0$

$2y^2 + 27y + 13 = 0$

Ways to solve a quadratic equations

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Solutions of a quadratic equation by completing the square

$$X^2 + 8X - 48 = 0$$

$$\therefore X^2 + 8X + 16 - 16 - 48 = 0$$

$$\therefore (X + 4)^2 - 64 = 0$$

$$\therefore (X + 4)^2 = 64$$

$$\therefore X + 4 = 8 \text{ or } X + 4 = -8$$

$$\therefore X = 4 \text{ or } X = -12$$

Solve these equations

$9y^2 - 12y + 2 = 0$

$2y^2 + 9y + 10 = 0$

$5x^2 = 4x + 7$

Ways to solve a quadratic equations

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: $x^2 + 10x + 2 = 0$ comparing with $ax^2 + bx + c = 0$

we get $a = 1,\ b = 10,\ c = 2,$

$\therefore\ b^2 - 4ac = (10)^2 - 4 \times 1 \times 2$

$= 100 - 8$

$= 92$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{92}}{2 \times 1}$$

$$x = \frac{-10 \pm \sqrt{4 \times 23}}{2}$$

$$= \frac{-10 \pm 2\sqrt{23}}{2}$$

$$= \frac{2(-5 \pm \sqrt{23})}{2}$$

$\therefore\ x = -5 \pm \sqrt{23}$

$\therefore\ x = -5 + \sqrt{23}$ or $x = -5 - \sqrt{23}$

. the roots of the given quadratic equation are $-5 + \sqrt{23}$ and $-5 - \sqrt{23}$.

Solutions of a quadratic equation by formula

Solve these equations

$9y^2 - 12y + 2 = 0$

$2y^2 + 9y + 10 = 0$

$5x^2 = 4x + 7$

Elaborate

(1) If α and β are roots of quadratic equation $ax^2 + bx + c = 0$,

(i) $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$

(ii) $\alpha + \beta = -\frac{b}{a}$ and $\alpha \times \beta = \frac{c}{a}$

(2) Nature of roots of quadratic equation $ax^2 + bx + c = 0$ depends on the value of $b^2 - 4ac$. Hence $b^2 - 4ac$ is called discriminant and is denoted by Greek letter Δ.

(3) If $\Delta = 0$ The roots of quadratic equation are real and equal.

If $\Delta > 0$ then the roots of quadratic equation are real and unequal.

If $\Delta < 0$ then the roots of quadratic equation are not real.

(4) The quadratic equation, whose roots are α and β is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Sign trick in quadratic equations .

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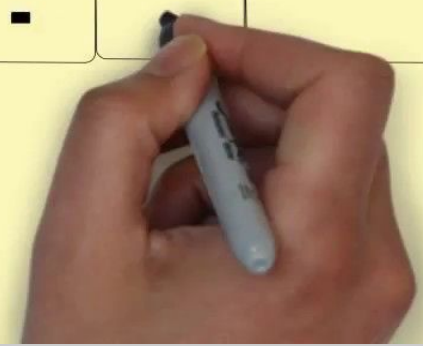
Sign of 'c'	Sign of 'b'	Sign of Roots
+	+	- -
+	-	+ +
-	+	+ -
-		



Now let us learn how to find sign of roots of any quadratic equation

1) When 'c' is +, then both roots have the same sign which is opposite of the sign of 'b'

2) When 'c' is -, then both roots have different signs & there is no need to see the sign of 'b'.

General form of quadratic equation:

$$ax^2 + bx + c = 0$$

Note how the signs change

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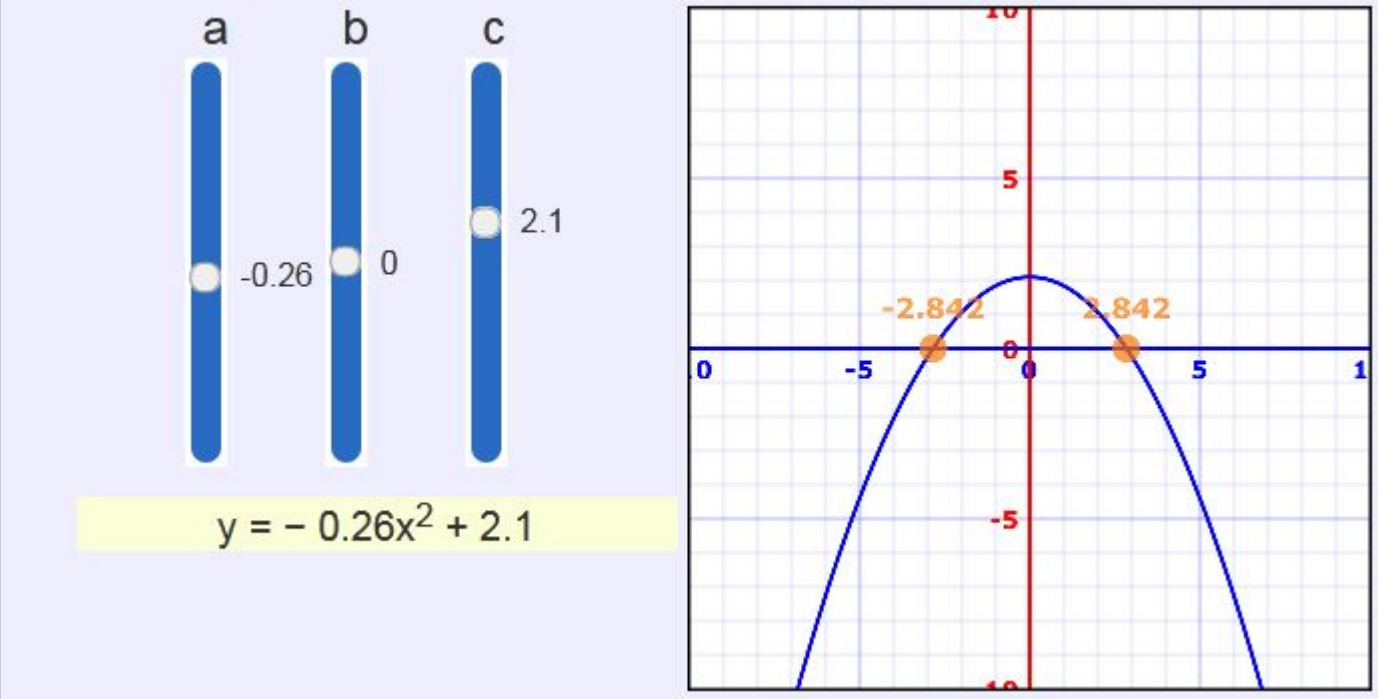
[Explore the Quadratic Equation] using the link.

See how changing the value of a, b and c changes the shape of the equations

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Thank you