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Arithmetic Progression

Grade 10, topic :3 (Maths 1)

Topics

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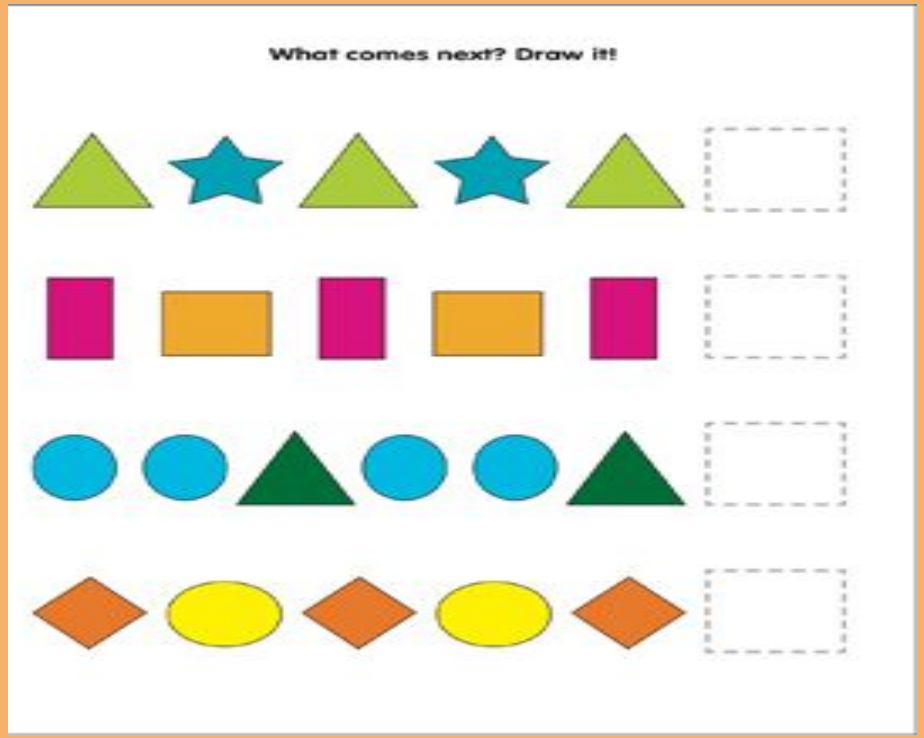
1. Sequence
2. Terms in a sequence
3. Arithmetic Progression
4. n^{th} term of an A.P.
5. Sum of n terms of an A.P.
6. Application in real life

Do you remember these ?



Do you remember these ?

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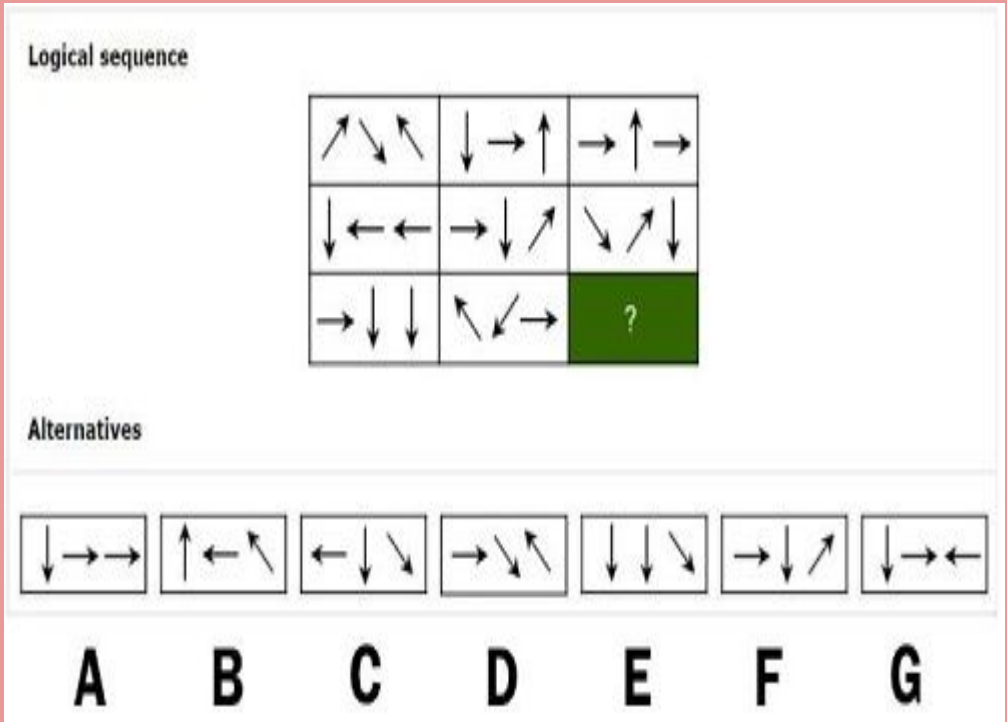

Can you solve these mentally ?

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a) $2, 4, 6, 8, 10, \ldots$

b) $1, 3, 5, 7, 9, \ldots$

c) $99, 199, 299, 399, 499, \ldots$

d) $3, -5, 7, -9, 11, \ldots$

e) $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \ldots$

f) $1, 4, 9, 16, 25, \ldots$

g) $0, 2, 6, 12, 20, \ldots$

h) $1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \ldots$

i) $6, 12, 20, 30, 42, \ldots$

j) $\frac{2}{3}, \frac{3}{2\times4}, \frac{4}{3\times5}, \frac{5}{4\times6}, \frac{6}{5\times7}, \ldots$

k) $0, \frac{1}{3}, 0, \frac{1}{3}, 0, \ldots$

l) $-\frac{1}{2}, \frac{2}{5}, -\frac{3}{8}, \frac{4}{11}, -\frac{5}{14}, \ldots$

Observe the changes

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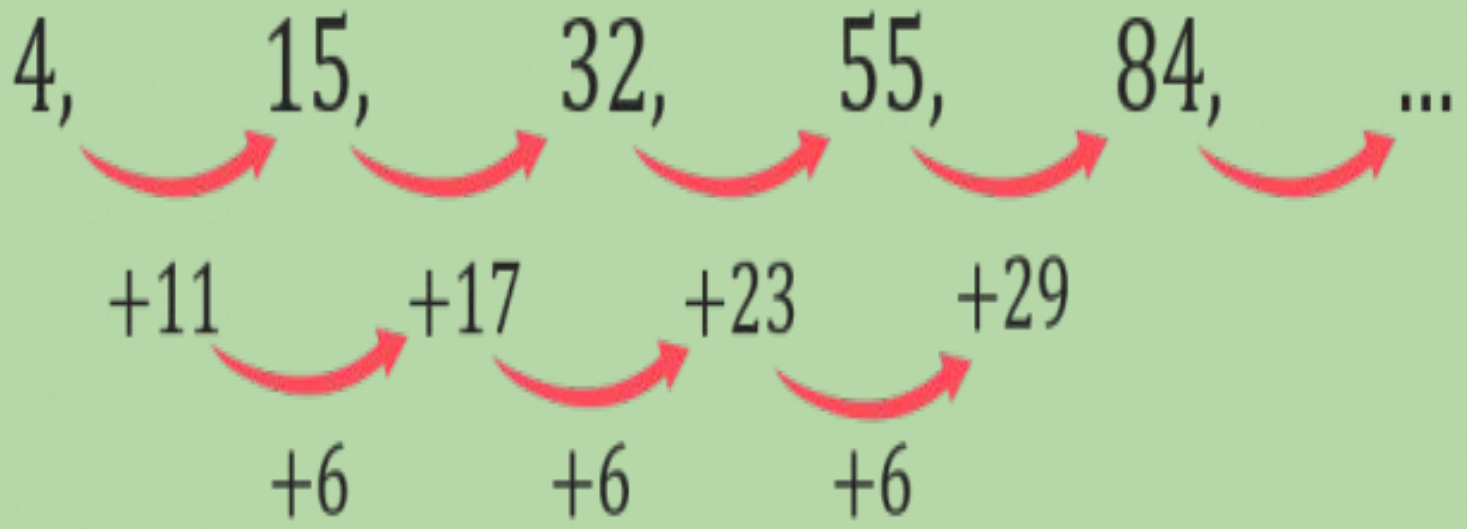

Now try to understand how the numbers Change

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Increasing Arithmetic Sequence	Decreasing Arithmetic Sequence
* Common difference is positive!	* Common difference is negative!
5 , 9 , 13 , 17 , ... +4 +4 +4	20 , 17 , 14 , 11 , ... -3 -3 -3

Observe the changes

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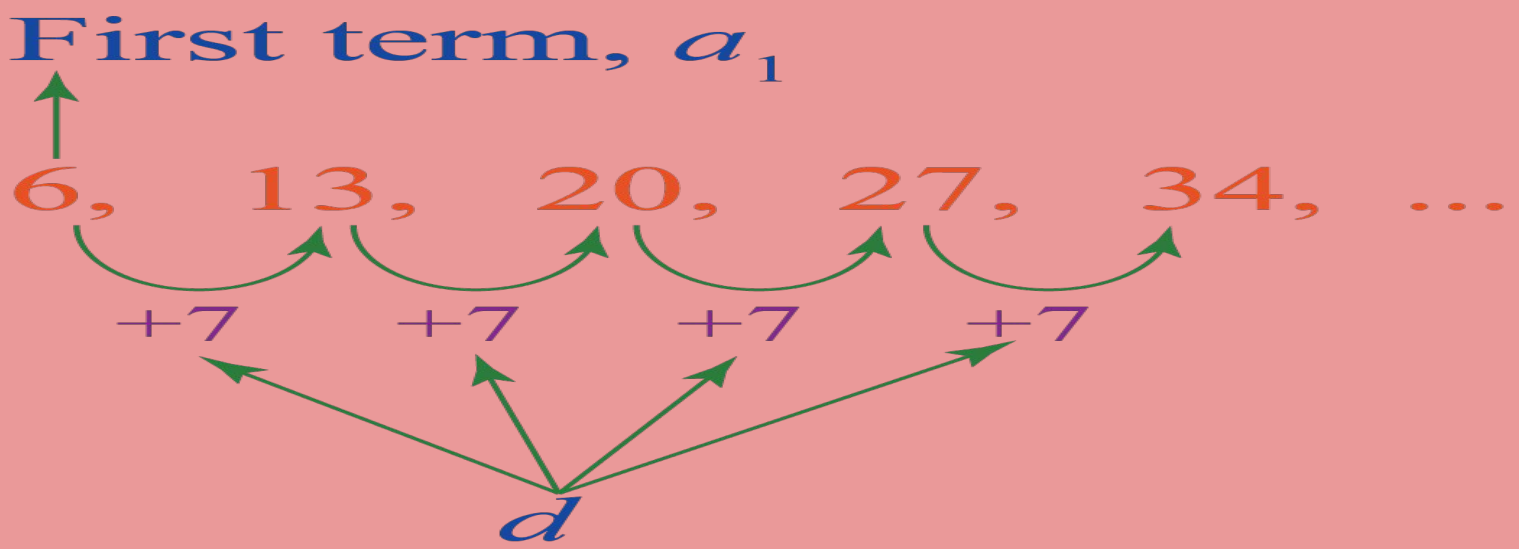
2, 6, 10, 14, ...

+4 +4 +4

Common difference



Common difference, $d = 7$

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Sequences

A sequence is a list of numbers that follow a certain rule.

Arithmetic Sequences

In an Arithmetic Sequence the difference between one term and the next is a constant.

$a, a+d, a+2d, a+3d, \ldots$

The n^{th} term, $a_n = a+(n-1)d$

Example: 1, 5, 9, 13, 17, ...

Terms in a sequence

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Sequences of Numbers

Definition A sequence $(x_1, x_2, x_3, \ldots)$ is a rule that assigns to each natural number n, the number x_n.

Examples

1. $\left(1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\right)$
2. $(1, 1.4, 1.41, 1.414, 1.4142, \ldots)$
3. $(1, -3, 5, -7, 9, \ldots)$

Definition of a sequence

A sequence is made up of terms, separated by commas:

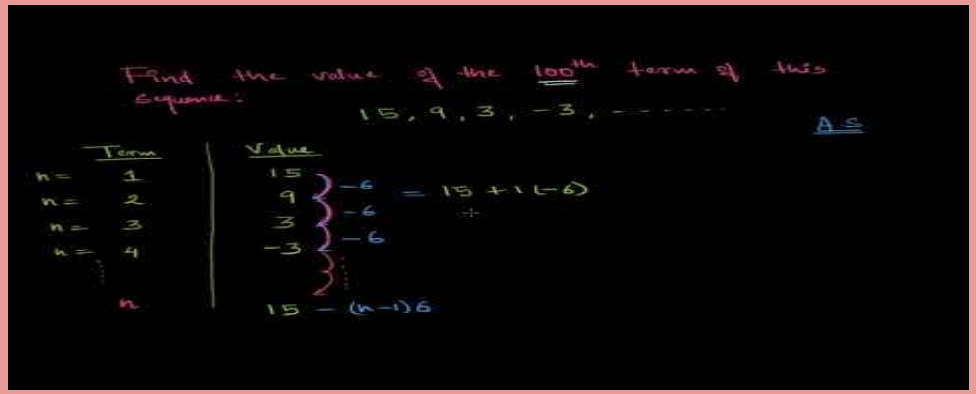
$1,\ 2,\ 3,\ 6,\ 11,\ 23,\ 47,\ 106,\ 235, \ldots$

$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, \ldots$

x_8 — Term number: 8 — Term

The position of a term in a sequence is denoted by a subscript.
Here 106 is the 8th term (x_8)

Terms in a sequence



To calculate terms in a sequence

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- **Find the first four terms of the sequence**

$$a_n = 3n - 2$$

$$a_1 = 3(1) - 2 = 1$$ First term

$$a_2 = 4$$ Second term

$$a_3 = 7$$ Third term

$$a_4 = 10$$ Fourth term

More examples

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Write the first six terms of the sequence.

a. $a_0 = 1,\ a_n = a_{n-1} + 4$ **b.** $a_1 = 1,\ a_n = 3a_{n-1}$

SOLUTION

a. $a_0 = 1$

$a_1 = a_0 + 4 = 1 + 4 = 5$

$a_2 = a_1 + 4 = 5 + 4 = 9$

$a_3 = a_2 + 4 = 9 + 4 = 13$

$a_4 = a_3 + 4 = 13 + 4 = 17$

$a_5 = a_4 + 4 = 17 + 4 = 21$

b. $a_1 = 1$

$a_2 = 3a_1 = 3(1) = 3$

$a_3 = 3a_2 = 3(3) = 9$

$a_4 = 3a_3 = 3(9) = 27$

$a_5 = 3a_4 = 3(27) = 81$

$a_6 = 3a_5 = 3(81) = 243$

Arithmetic sequences ....

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Arithmetic sequences

An **arithmetic sequence** or **arithmetic progression (AP)**, is a sequence whose terms go up or down by constant steps i.e. there is a **common difference**.

Examples:

(i) 5, 7, 9, 11, 13, ...

(ii) 88, 78, 68, 58, 48, ...

- The first term of an **AP** is denoted by **a**: $u_1 = a$
- The **common difference** is denoted by **d**: $u_{n+1} = u_n + d$
- Formula for the n^{th} term of AP is $a + (n - 1)d$
- n^{th} term: $u_n = a + (n - 1)d$ **or** $a + d(n - 1)$

Watch the videos

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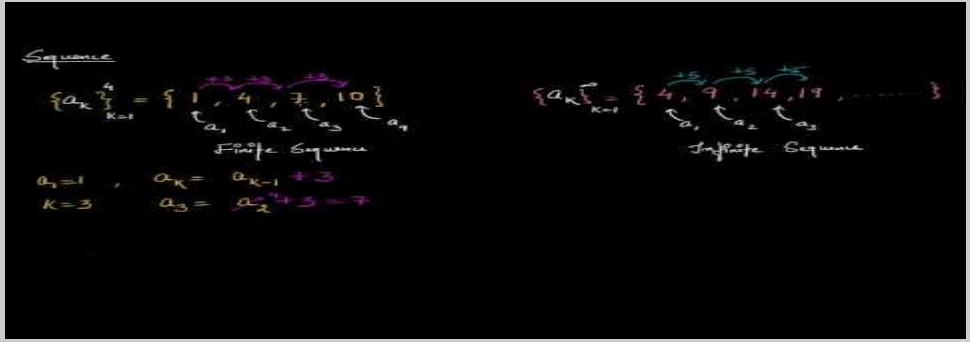

Sequence : A set of numbers where the numbers are arranged in a definite order.

अनुक्रम: संख्याओं का एक समूह जहां संख्याओं को एक निश्चित क्रम में व्यवस्थित किया जाता है।

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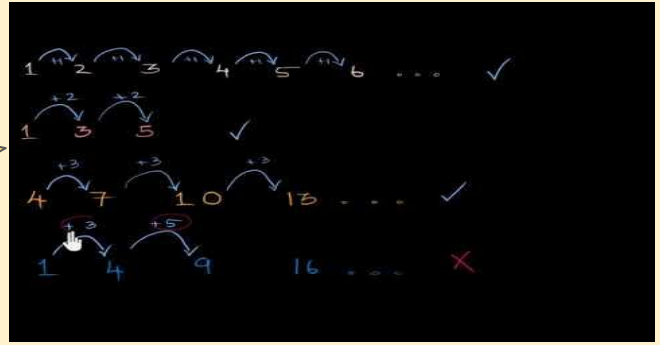
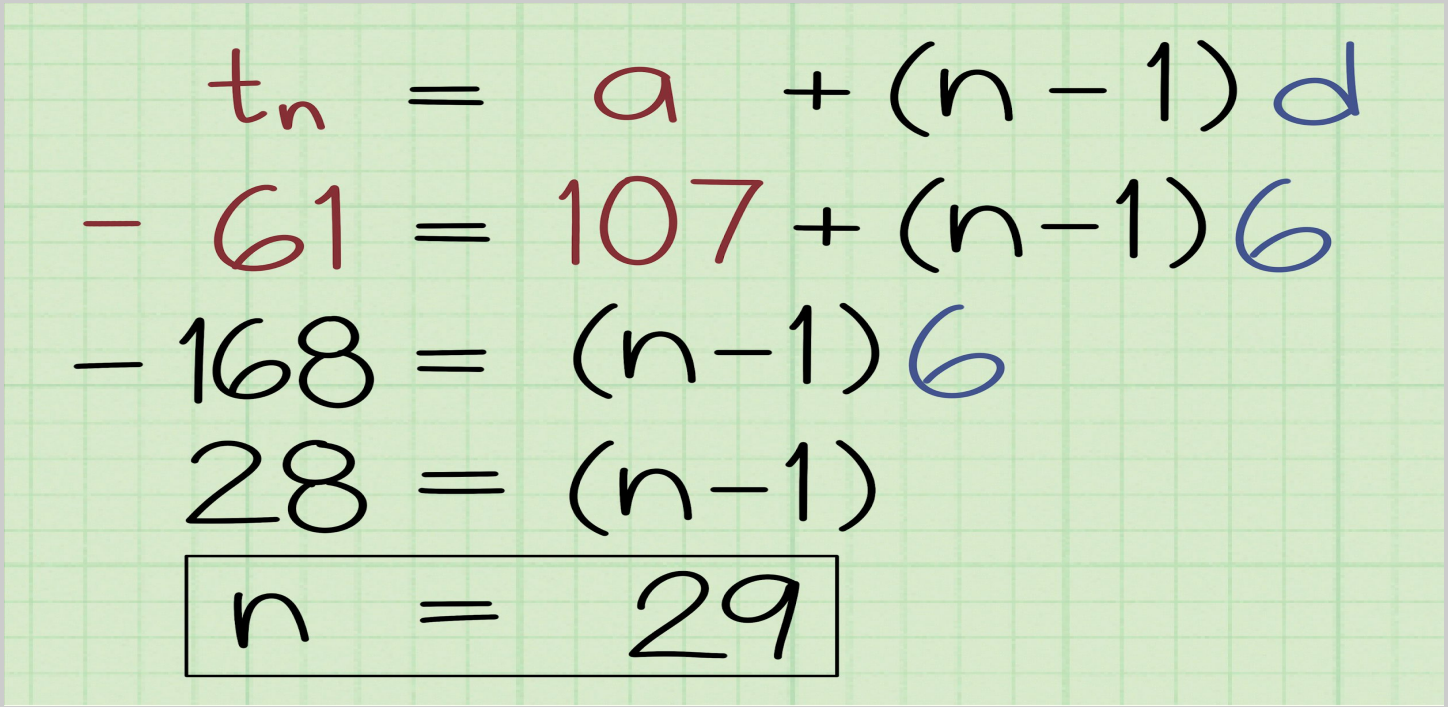
What is arithmetic progression ?



How many terms in a sequence ?



To calculate the nth term in the sequence

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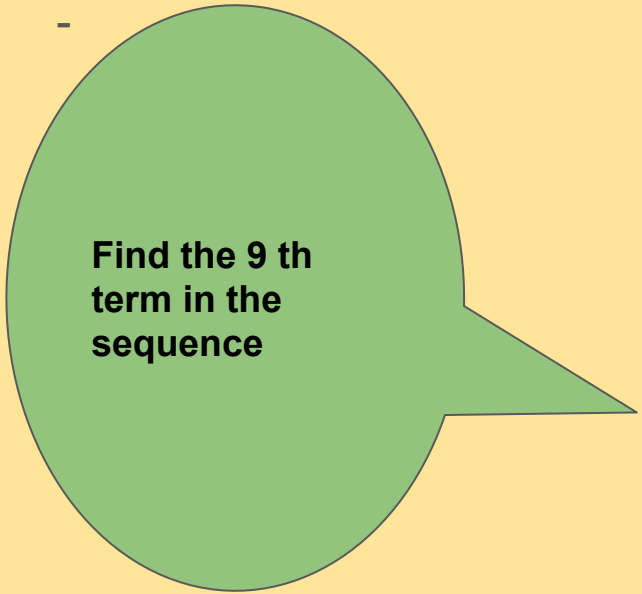
How to find the Nth term of an AP.एक एपी के Nth term को खोजने के लिए।

To calculate a particular term in an AP



21, 17, 13, 9, 5…

$$a_n = a_1 + (n+1)d$$
$$a_n = 21 + (n+1)(-4)$$
$$a_n = 21 + -4n - 4$$
$$a_n = -4n + 17$$
$$a_9 = -4(9) + 17$$
$$a_9 = -19$$

Sum of n terms of AP:

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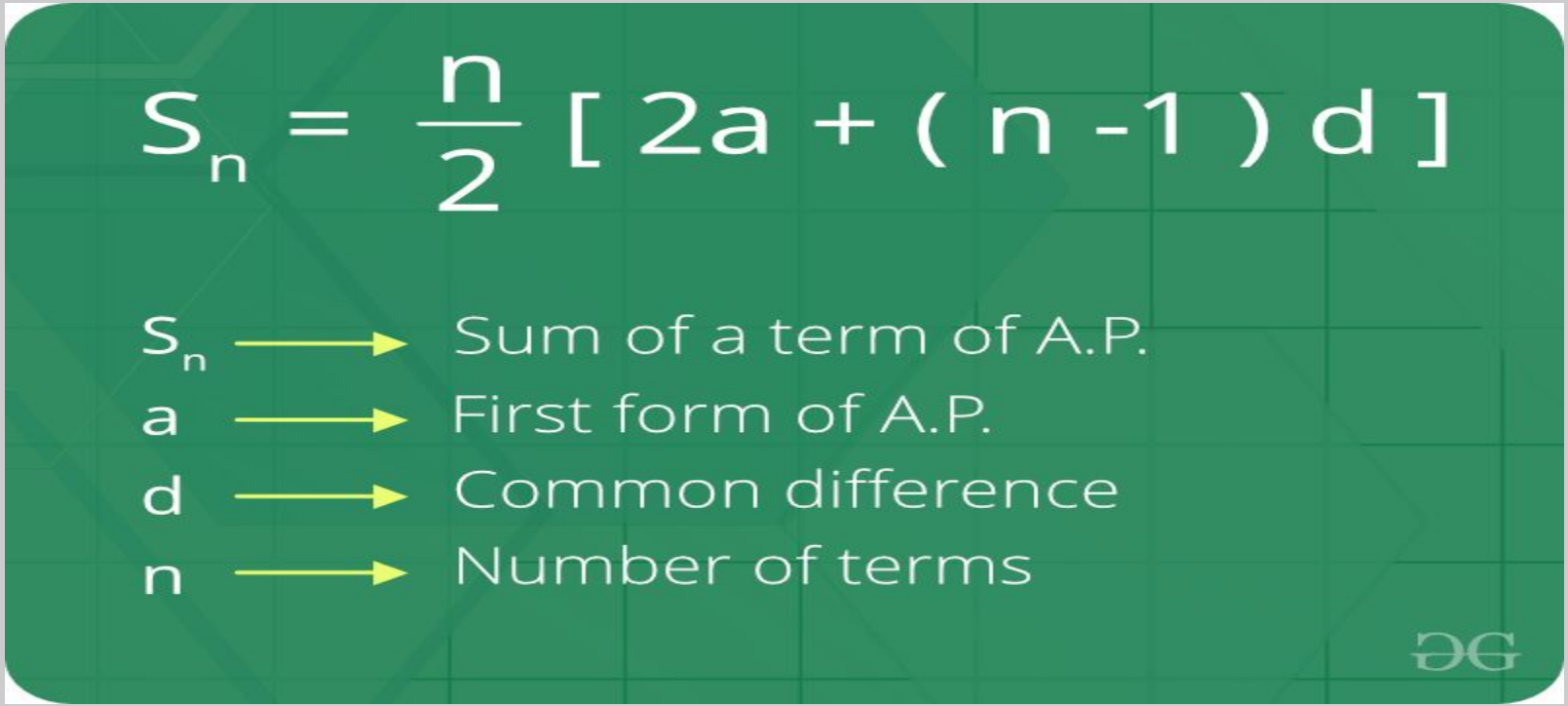
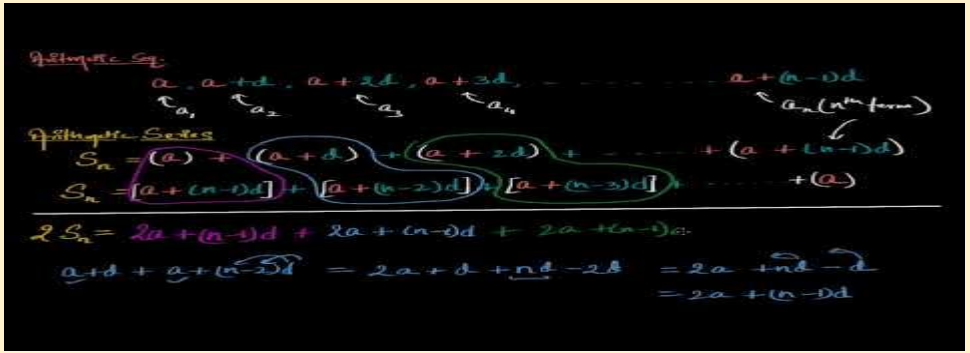


Explain and Elaborate

Sum of n terms in the AP series

एपी श्रृंखला में n terms का योग



Can we fill up the blanks orally ?

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5. Complete the following activity to find the sum of natural numbers between 1 and 140 which are divisible by 4.

Between 1 and 140, natural numbers divisible by 4

↓

4, 8, ... , 136

↓

How many numbers ? $\therefore$ $n =$ ☐

↓

$n =$ ☐ , $a =$ ☐ , $d =$ ☐

↓

$t_n = a + (n-1)d$

↓

$136 =$ ☐ $+ (n - 1) \times$ ☐

→ $n =$ ☐ → $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{\square} = \frac{\square}{2}$ [ ] = ☐

Sum of numbers from 1 to 140, which are divisible by 4 = ☐

An example

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Example #2: Find the sum of the first 60 terms of the following series.

$$9+14+19+24+\ \ldots\ +289+294+299+304$$

Step #1: Identify the variables.

$$n=60,\ \ a_1=9,\ \ a_{60}=304$$

Step #2: Substitute and evaluate.

$$S_n=\frac{n}{2}(a_1+a_n)$$

$$S_n=\frac{60}{2}(9+304)=30\times 313=9390$$

Formulae that you need to remember

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$t_n = a + (n-1)d$
$S_n = \frac{n}{2}[2a + (n-1)d]$
$S_n = \frac{n}{2}[a + l]$

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Thank you