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Circle

Grade 10 , Mathematics II (Topic 3)

This is something interesting

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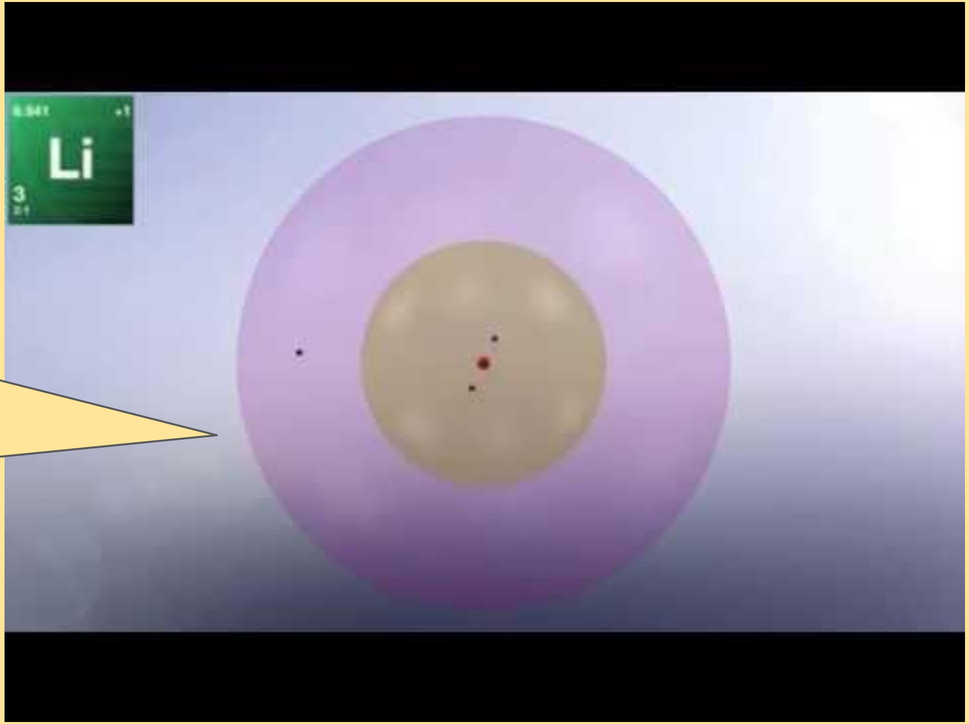
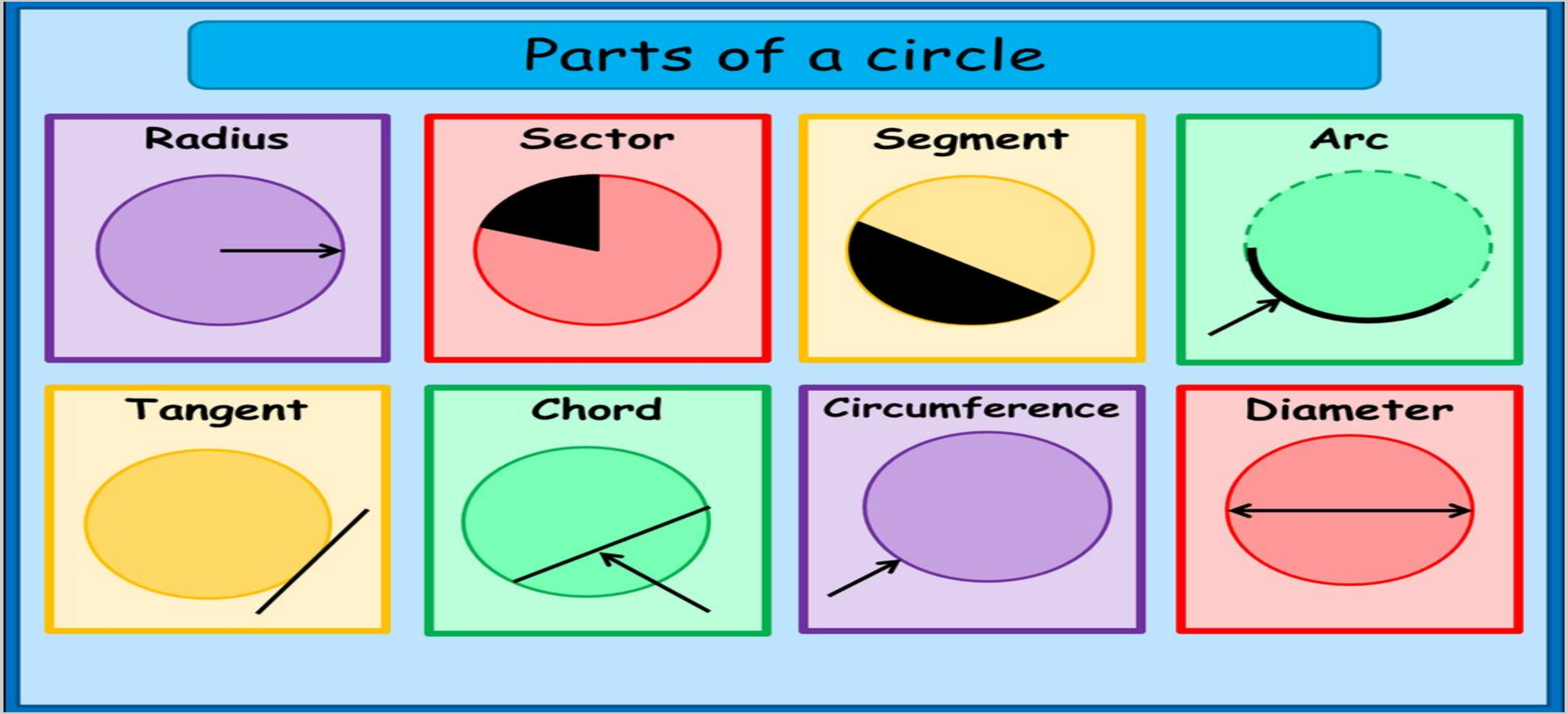
Did you know this ?

क्या आप यह जानते थे ?

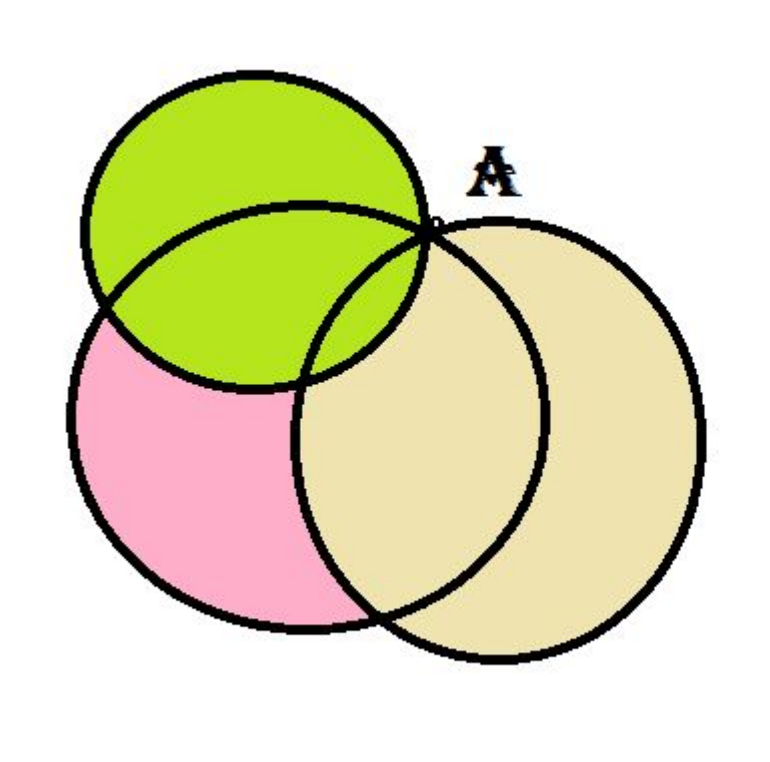
तुला हे माहित आहे का?



Explore : Previous knowledge



Circle passing through points



In this adjacent diagram , only three circles are passing through Point A . There can be innumerable circles which can pass through A .

इस चित्र में केवल तीन वृत्त बिंदु A से गुजर रहे हैं। असंख्य वृत्त हो सकते हैं जो A से होकर गुजर सकते हैं।

या समीप आकृतीमध्ये, केवळ तीन मंडळे पॉईंट एमधून जात आहेत. ए मधून पुढे जाणारे असंख्य मंडळे असू शकतात

Secant and tangent

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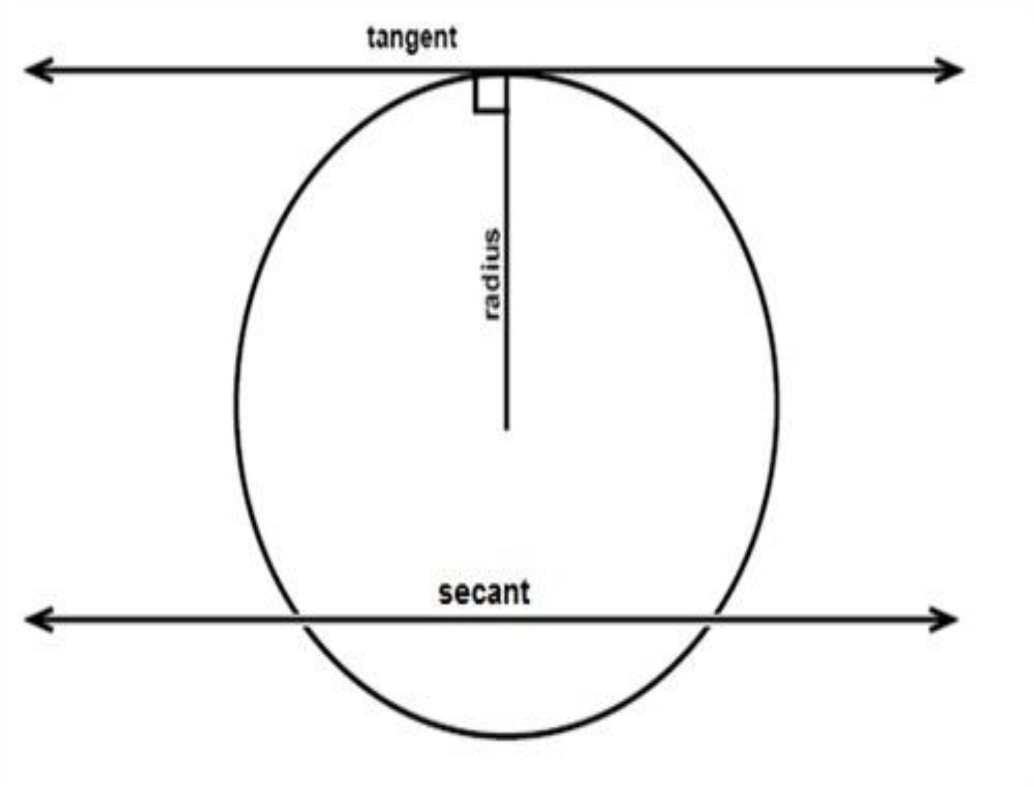


Tangent touches only one point of the circle .

The secant passes through two points of the circumference .

स्पर्शरेखा वृत्त के केवल एक बिंदु को छूती है।
__________ परिधि के दो बिंदु से होकर गुजरती है।
टॅन्जेंट वर्तुळाच्या केवळ एका बिंदूला स्पर्श करते.
सेन्टेंट परिघाच्या दोन बिंदूतून जातो.

Tangent Theorem

Tangent theorem

Theorem : A tangent at any point of a circle is perpendicular to the radius at the point of contact.

There is an indirect proof of this theorem.

For more information

Given : Line l is a tangent to the circle with centre O at the point of contact A.

To prove : line $l \perp$ radius OA.

Proof : Assume that, line l is not perpendicular to seg OA.

Suppose, seg OB is drawn perpendicular to line l.

Of course B is not same as A.

Now take a point C on line l such that A-B-C and BA = BC .

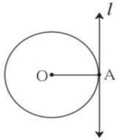


Fig. 3.10

Now in, Δ OBC and Δ OBA

seg BC $\cong$ seg BA .... (construction)

$\angle$ OBC $\cong$ $\angle$ OBA .... (each right angle)

seg OB $\cong$ seg OB

$\therefore$ Δ OBC $\cong$ Δ OBA .... (SAS test)

$\therefore$ OC = OA

But seg OA is a radius.

$\therefore$ seg OC must also be radius.

$\therefore$ C lies on the circle.

That means line l intersects the circle in two distinct points A and C.

But line l is a tangent. .... (given)

$\therefore$ it intersects the circle in only one point.

Our assumption that line l is not perpendicular to radius OA is wrong.

$\therefore$ line $l \perp$ radius OA.

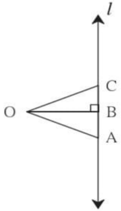
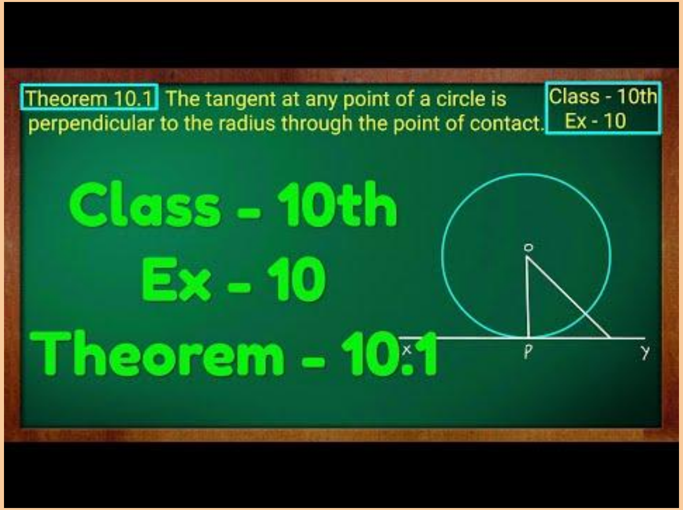


Fig. 3.11

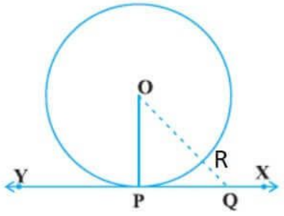


Converse of Tangent Theorem

The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Given: A circle with center O.

With tangent XY at point of contact P.



To prove: OP ⊥ XY

Proof: Let Q be point on XY

Connect OQ

Suppose it touches the circle at R

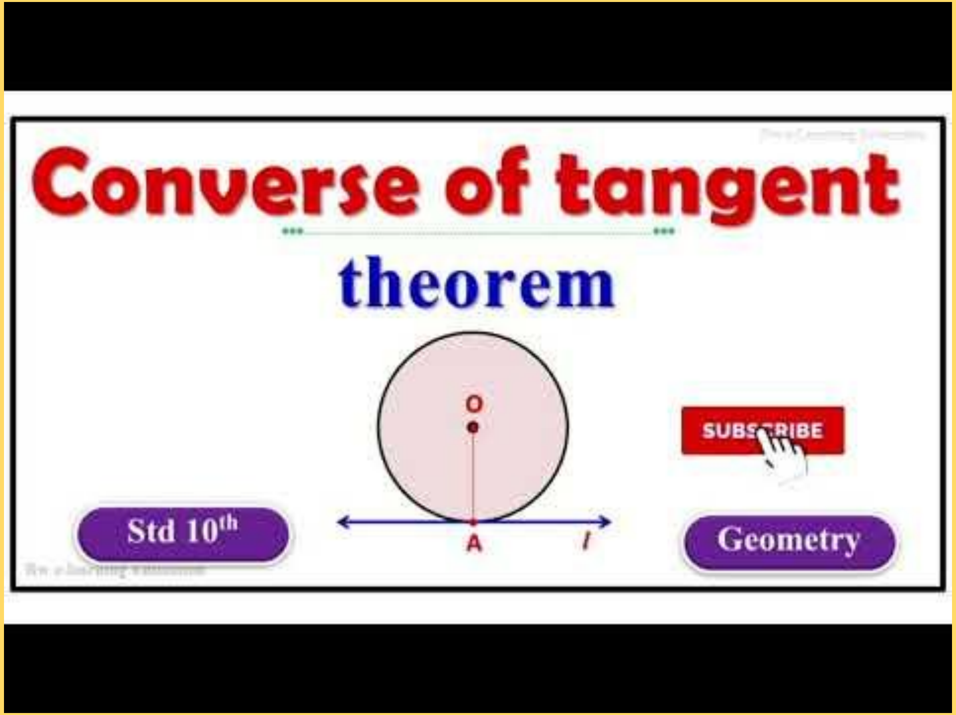
Hence,

$OQ > OR$

$OQ > OP$ *(as OP = OR radius)*

Same will be the case with all other points on circle

Hence, OP is the smallest line that connects XY



Tangent Segment theorem

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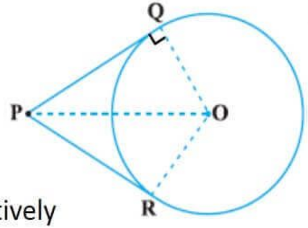
The lengths of tangents drawn from an external point to a circle are equal.



Given: Let circle be with centre O

and P be a point outside circle

PQ and PR are two tangents to circle

intersecting at point Q and R respectively

To prove: Lengths of tangents are equal

i.e. PQ = PR

Construction: Join OQ , OR and OP

Proof: As PQ is a tangent

$OQ \perp PQ$ *(Tangent at any point of circle is perpendicular to the radius through point of contact)*

So, $\angle OQP = 90°$

Hence Δ OQP is right triangle

https://www.khanacademy.org/math/in-in-class-10-math-cbse-hindi/xf0551d6b19cc0b04:circles/xf0551d6b19cc0b04:number-of-tangents-from-a-point-to-a-circle/v/proof-segments-tangent-to-circle-from-outside-point-are-congruent-hindi

(proof in Hindi)

Explain : Theorem of touching circles

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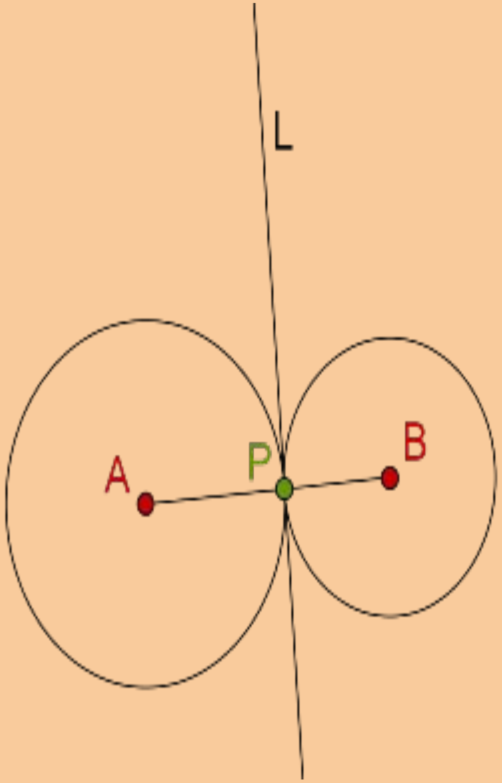
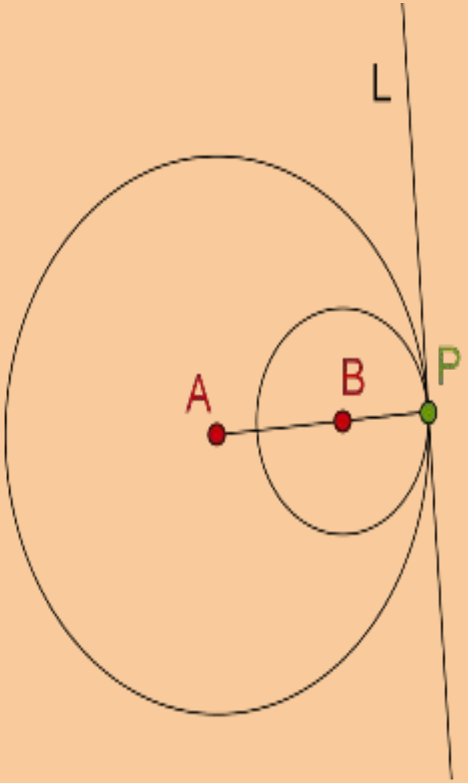
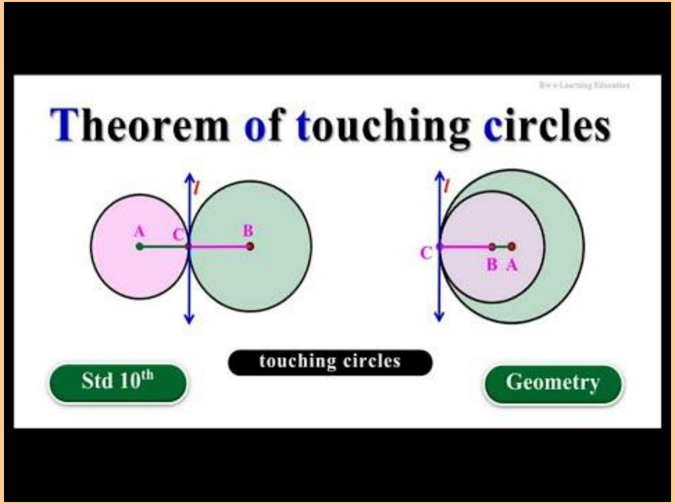
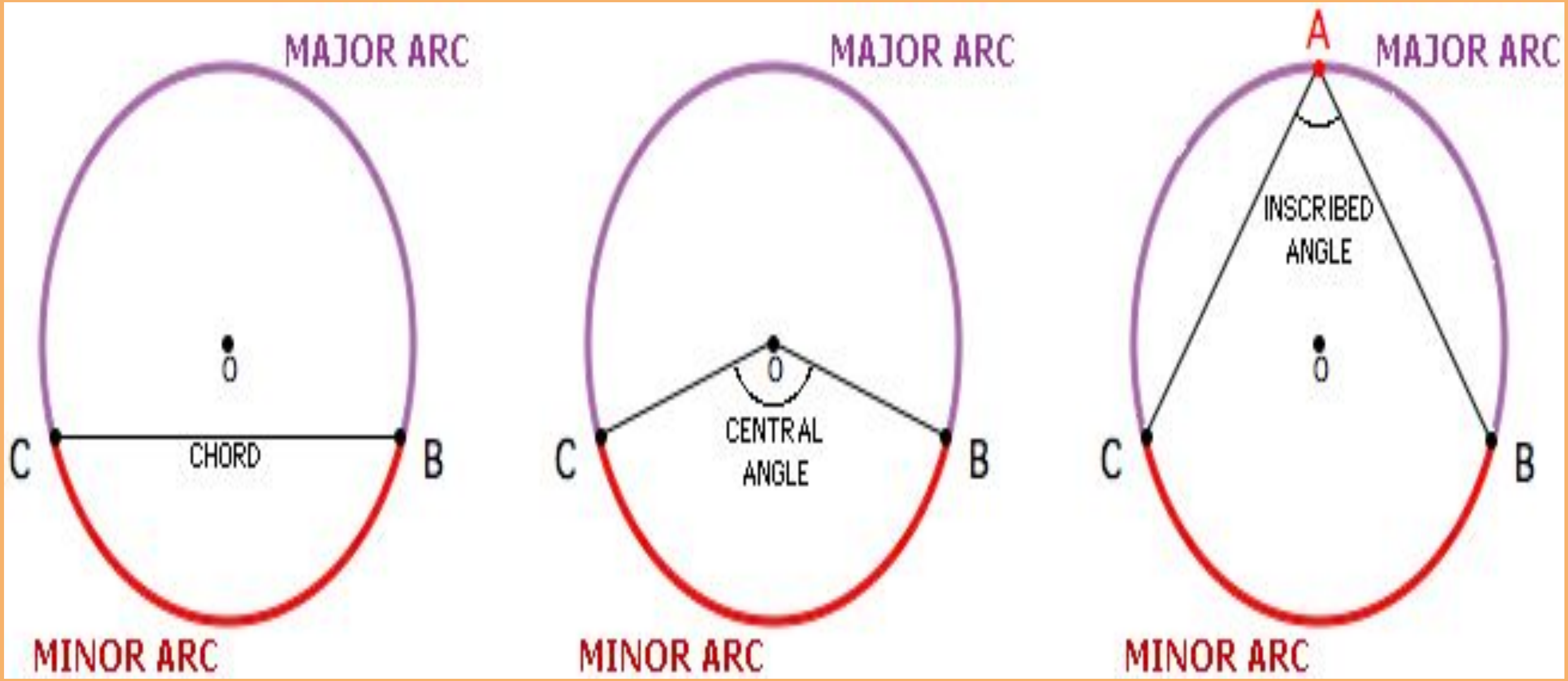






Explain : Major arc, Minor arc , Central angle



Arcs of a circle

Arc of a circle

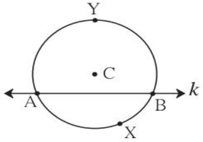


Fig. 3.29

A secant divides a circle in two parts. Any one of these two parts and the common points of the circle and the secant constitute an **arc of the circle.**

The points of intersection of circle and secant are called the end points of the arcs.

In figure 3.29, due to secant k we get two arcs of the circle with centre C– arc AYB, arc AXB.

If the centre of a circle is on one side of the secant then the arc on the side of the centre is called **'major arc'** and the arc which is on the other side of the centre is called **'minor arc'**. In the figure 3.29 arc AYB is a major arc and arc AXB is a minor arc. If there is no confusion then the name of a minor arc is written using its end points only. For example, the arc AXB in figure 3.29, is written as arc AB.

Here after, we are going to use the same convention for writing the names of arcs.

Central angle

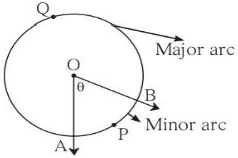


Fig. 3.30

When the vertex of an angle is the centre of a circle, it is called a central angle. In the figure 3.30, O is the centre of a circle and $\angle$AOB is a central angle.

Like secant, a central angle also divides a circle into two arcs.

Congruent arcs

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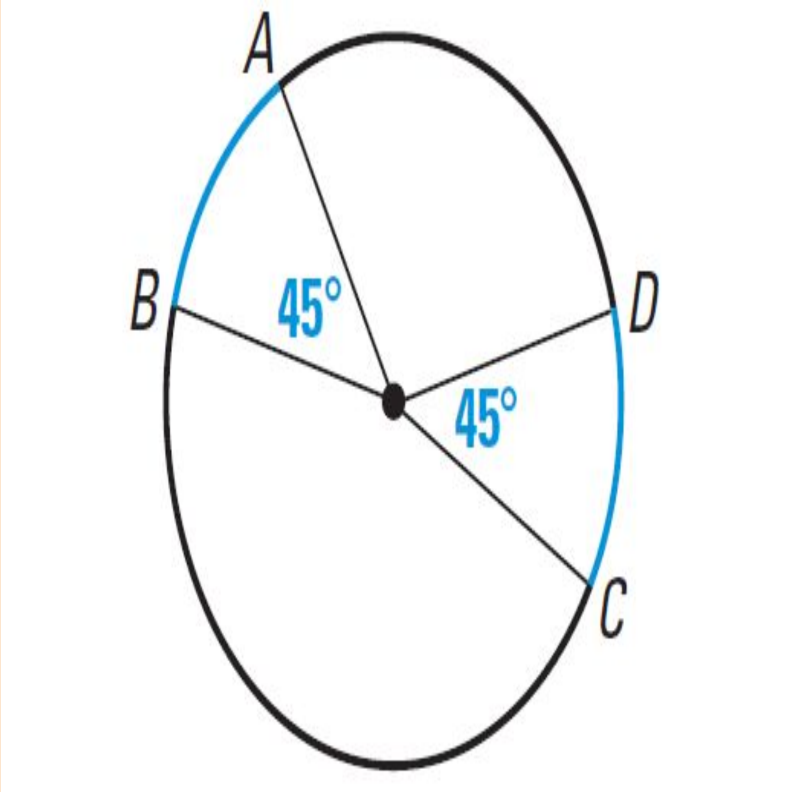


Congruent Arcs

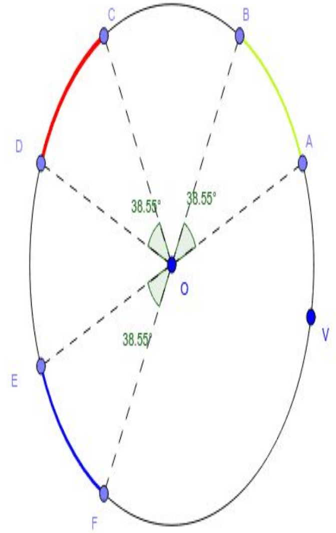
Two arc $\widehat{AB}$ and $\widehat{CD}$ are a part of **circumference circle** "O".

These arcs are congruent if and only if the two central angles $A\hat{O}B$ e $C\hat{O}D$ that intercept them are congruent.

$\widehat{AB} \equiv \widehat{CD} \leftrightarrow A\hat{O}B \equiv C\hat{O}D$

In the next structure $\widehat{AB}$, $\widehat{CD}$ and $\widehat{EF}$ are congruent.

Note that the angles $A\hat{O}B = 38.55°$, $C\hat{O}D = 38.55°$ e $E\hat{O}F = 38.55°$ are congruent.



Explain : Measure of arcs

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Property of sum of measures of arcs

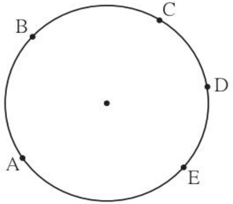

Fig. 3.32

In figure 3.32, the points A, B, C, D, E are concyclic. With these points many arcs are formed. There is one and only one common point C to arc ABC and arc CDE. So measure of arc ACE is the sum of measures of arc ABC and arc CDE.

$m(\text{arc ABC}) + m(\text{arc CDE}) = m(\text{arc ACE})$

But arc ABC and arc BCE have many points in common. [All points on arc BC.]

So $m(\text{arc ABE}) \neq m(\text{arc ABC}) + m(\text{arc BCE})$.

Theorem : The chords corresponding to congruent arcs of a circle (or congruent circles) are congruent.

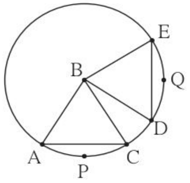

Fig. 3.33

Given : In a circle with centre B arc APC ≅ arc DQE

To Prove : Chord AC ≅ chord DE

Proof : (Fill in the blanks and complete the proof.)

In Δ ABC and Δ DBE,

side AB ≅ side DB (........)

side .... ≅ side .... (........)

∠ABC ≅ ∠DBE measures of congruent arcs

∴ Δ ABC ≅ Δ DBE (.......)

∴ chord AC ≅ chord DE (......)

Theorem & converse(congruent arcs)

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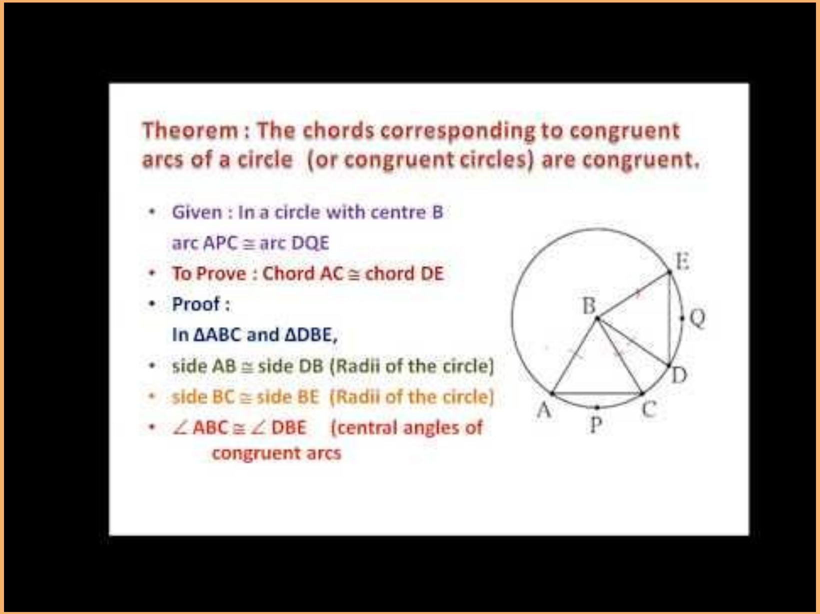
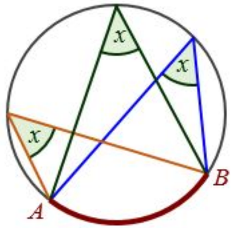
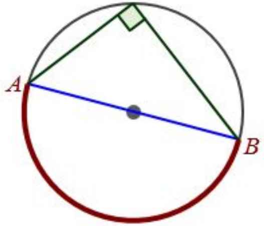
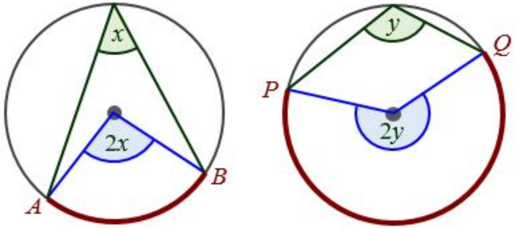
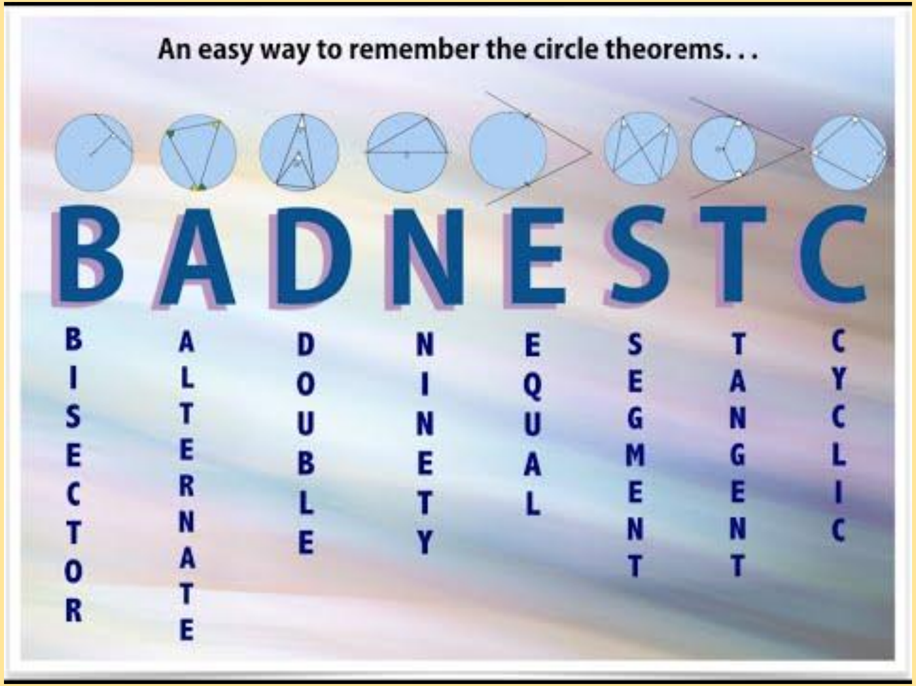
Theorem : The chords corresponding to the congruent arcs of a circle are congruent and converse .

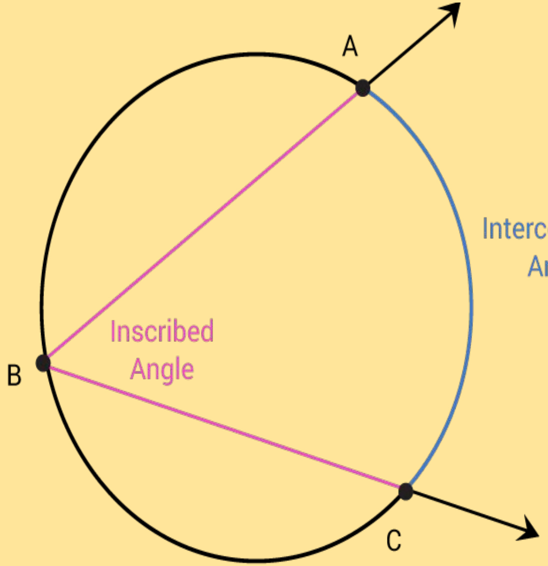


Angles in a circle

Angles In A Circle	
Inscribed angles subtended by the same arc are equal.	
Angles subtended by the diameter (or semi-circle) is 90°.	
Central angle is twice any inscribed angle subtended by the same arc.	

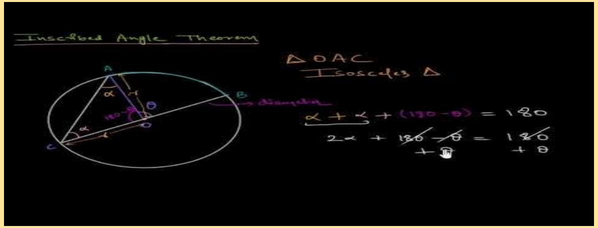


Inscribed angle theorem .



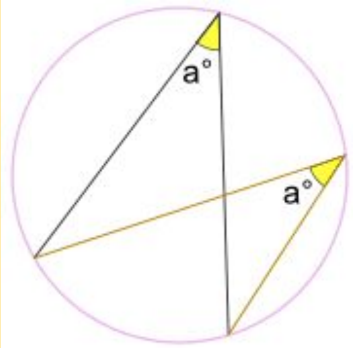
$\angle ABC = \frac{1}{2}\overset{\frown}{AC}$

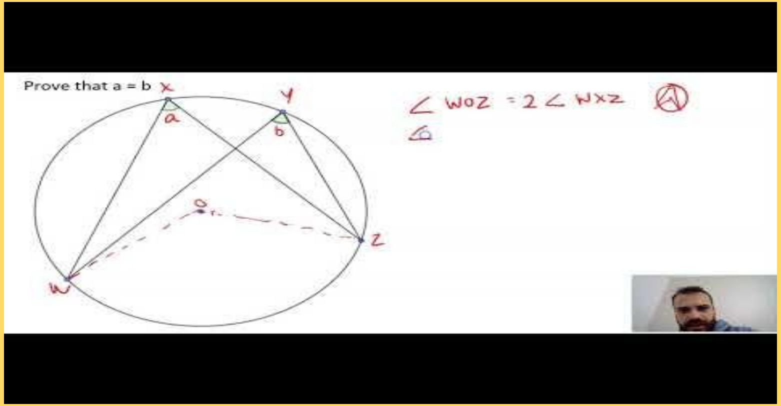
The measure of an inscribed angle is half of the measure of the arc intercepted by it .

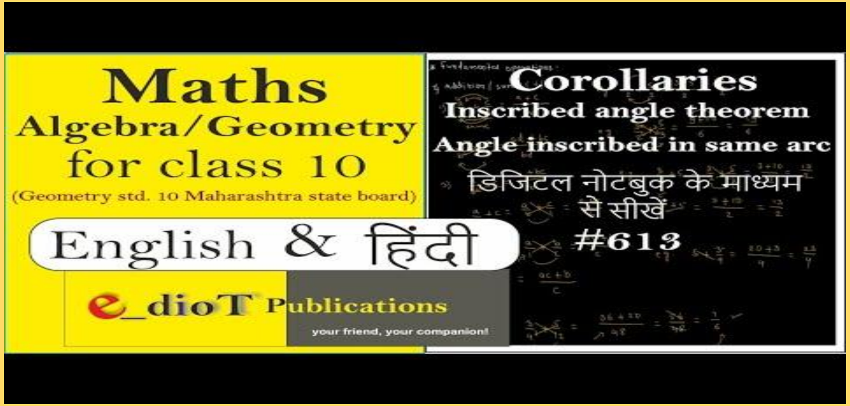


Angles inscribed in the same arc

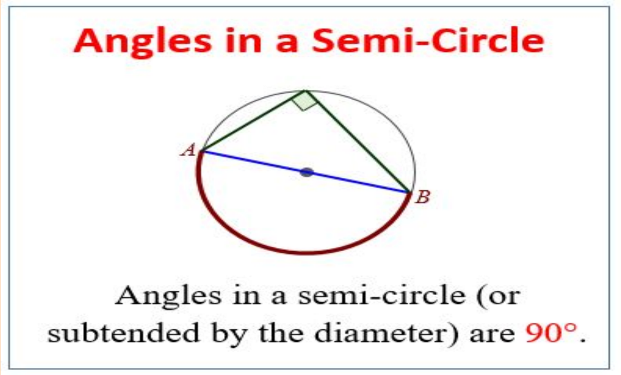
Angles inscribed in the same arc are congruent .

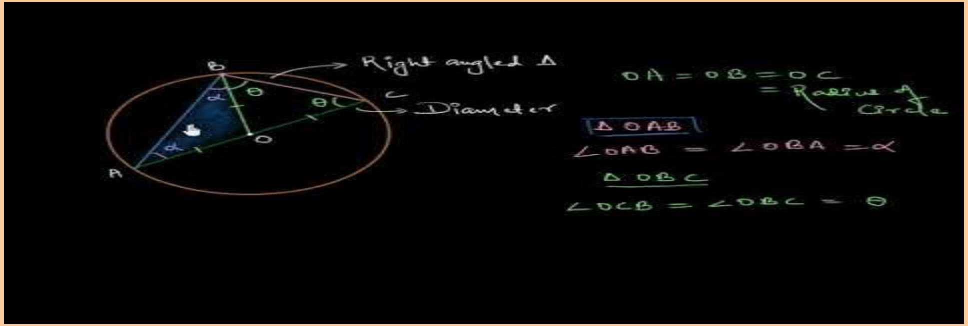


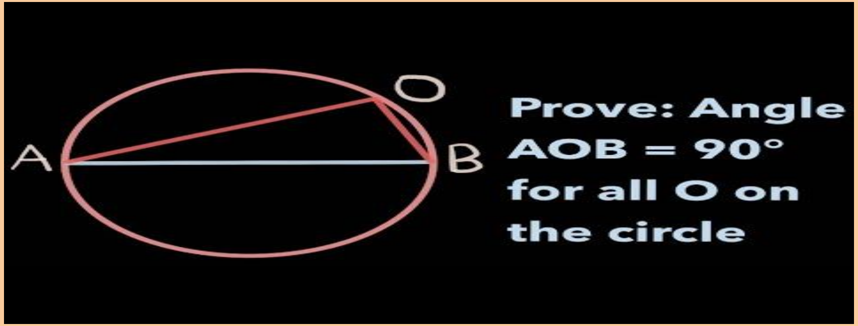




Angle inscribed in a Semicircle



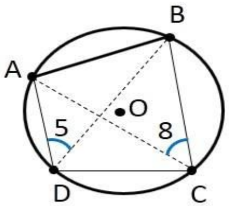




Explain : Cyclic quadrilateral

Chord AB

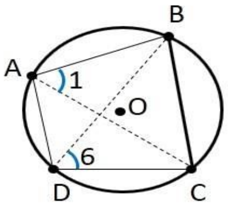
Angles in same segment are equal.



$\angle 5 = \angle 8$ ...(1)

Chord BC

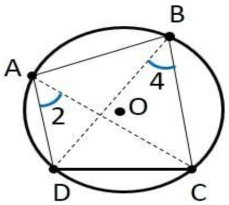
Angles in same segment are equal.



$\angle 1 = \angle 6$ ...(2)

Chord CD

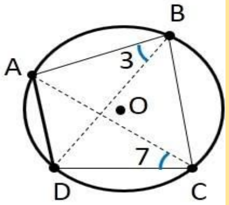
Angles in same segment are equal.



$\angle 2 = \angle 4$ ...(3)

Chord AD

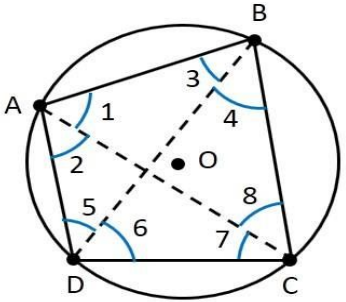
Angles in same segment are equal.

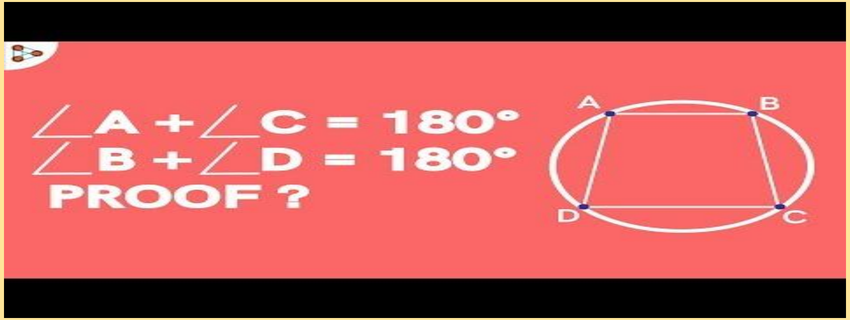


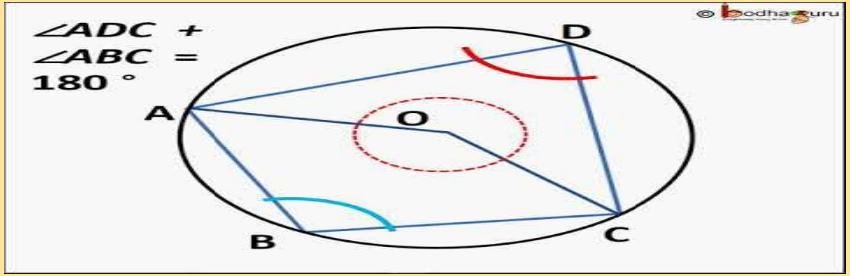
$\angle 7 = \angle 3$ ...(4)

By angle sum property of quadrilateral

$\angle A + \angle B + \angle C + \angle D = 360°$







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