kotak Education Foundation

Similarity

Grade 10 : Topic : 1 ,Mathematics II

Topics

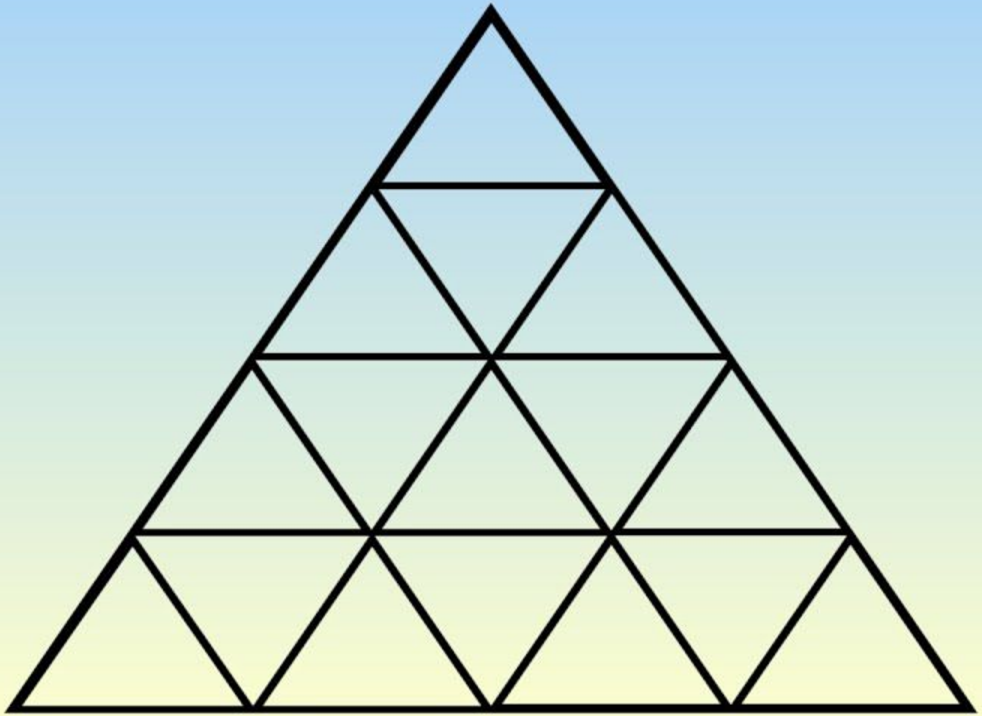
kotak Education Foundation

- Ratios of area of two triangles (condition 1 & 2)
- Basic Proportionality theorem
- Converse of basic proportionality theorem
- Tests of similarity of triangles
- Theorem of areas of similar triangles .

How many triangles do you see ?

kotak Education Foundation

HOW MANY TRIANGLES DO YOU SEE?

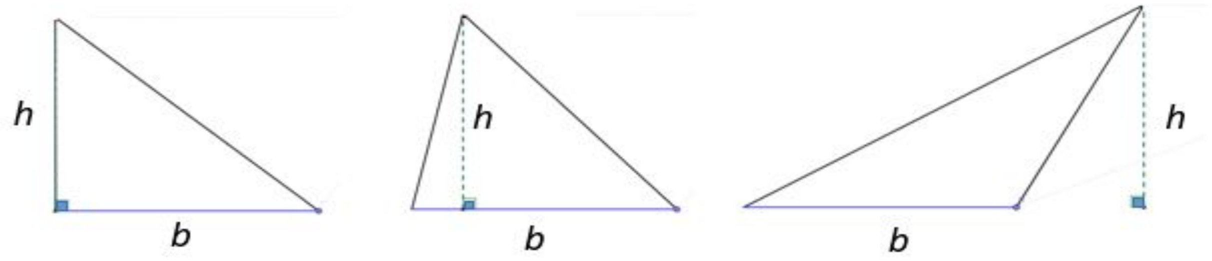


Here we are

Area of Triangle

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

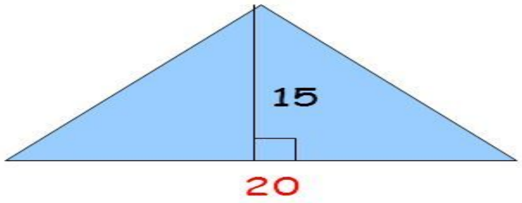
$$A = \frac{1}{2}bh$$



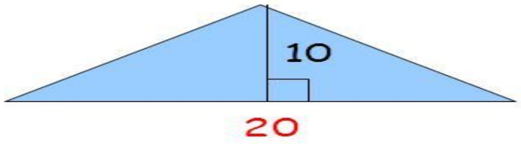
Comparing Areas of Triangles

If two triangles have equal bases, then the ratio of their areas equals the ratio of their heights.

$A = \frac{1}{2}(20)(15) = 150$



$A = \frac{1}{2}(20)(10) = 100$

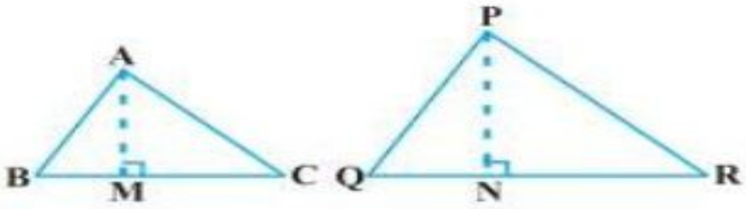


Ratio of heights: $\frac{15}{10} = \frac{3}{2}$

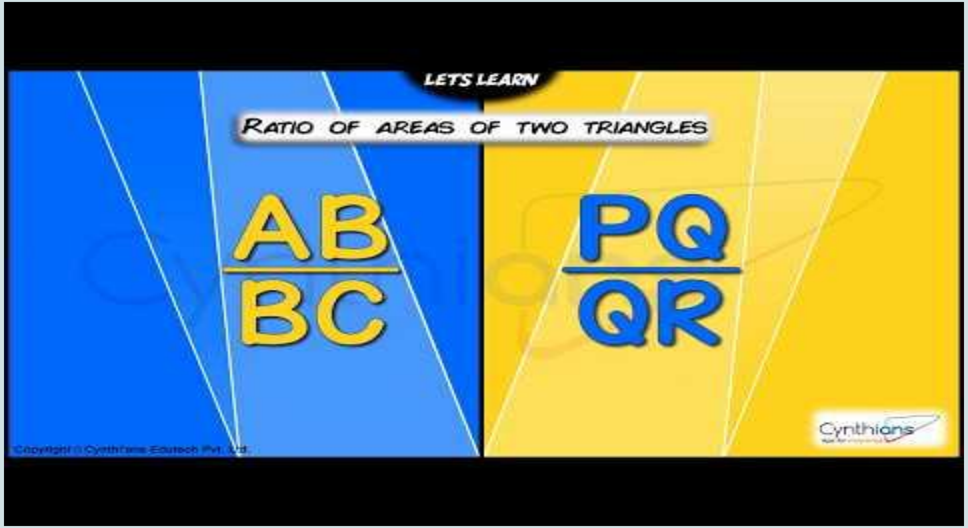
Ratio of areas: $\frac{150}{100} = \frac{3}{2}$

Area Theorem

- The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides
- It proves that $\frac{ar\,(ABC)}{ar\,(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$ in the figure given below

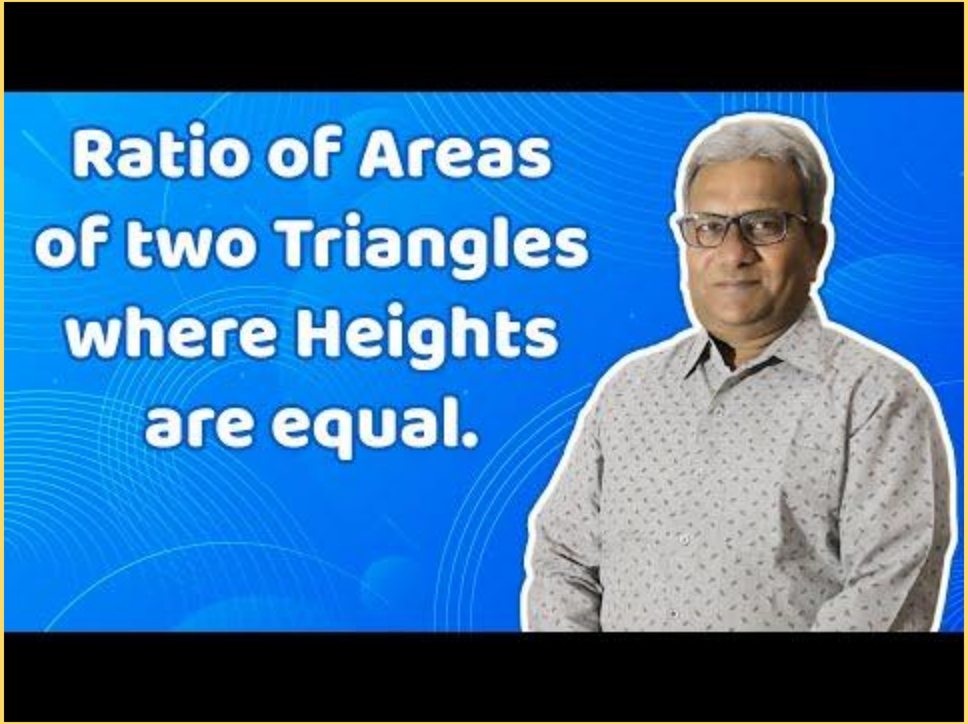


Ratios of areas of two triangles

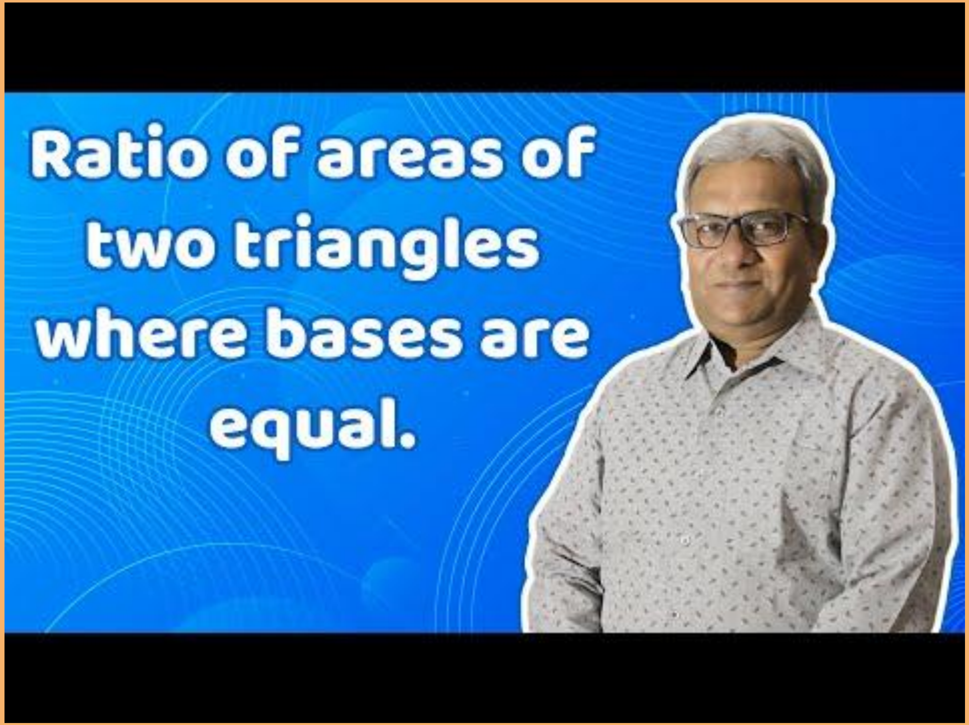


Condition 1 (if the heights of the triangles are equal)

kotak Education Foundation



Condition 2 : If the bases of both the triangles are equal

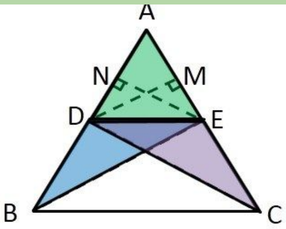


Basic Proportionality theorem was introduced by a famous Greek Mathematician, Thales, hence it is also called **Thales Theorem**. According to him, for any two equiangular triangles, the ratio of any two corresponding sides is always the same. Based on this concept, he gave theorem of basic proportionality (BPT). This concept has been introduced in similar triangles. If two triangles are similar to each other then,

- i) Corresponding angles of both the triangles are equal
- ii) Corresponding sides of both the triangles are in proportion to each other

teachoo.com

Now,



$$\text{ar (ADE)} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times AD \times EN \quad \text{...(1)}$$

$$\text{ar (BDE)} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times DB \times EN \quad \text{...(2)}$$

Divide (1) and (2)

$$\frac{\text{ar (ADE)}}{\text{ar (BDE)}} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN}$$

$$\frac{\text{ar (ADE)}}{\text{ar (BDE)}} = \frac{AD}{DB} \quad \text{...(A)}$$

$$\text{ar (ADE)} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times AE \times DM \quad \text{...(3)}$$

$$\text{ar (DEC)} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

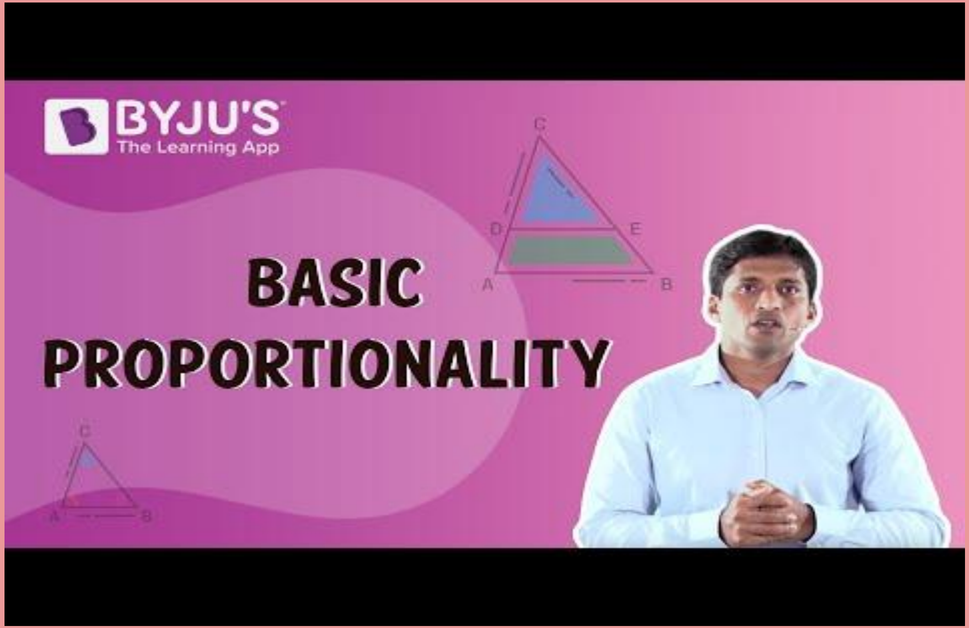
$$= \frac{1}{2} \times EC \times DM \quad \text{...(4)}$$

Divide (3) and (4)

$$\frac{\text{ar (ADE)}}{\text{ar (DEC)}} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM}$$

$$\frac{\text{ar (ADE)}}{\text{ar (DEC)}} = \frac{AE}{EC} \quad \text{...(B)}$$

Basic Proportionality Theorem

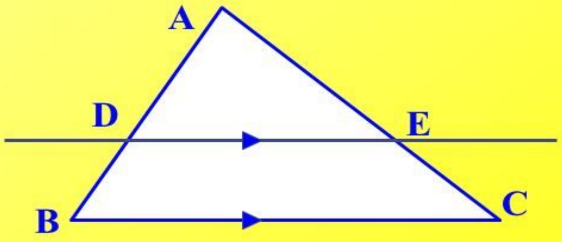


Converse of BPT

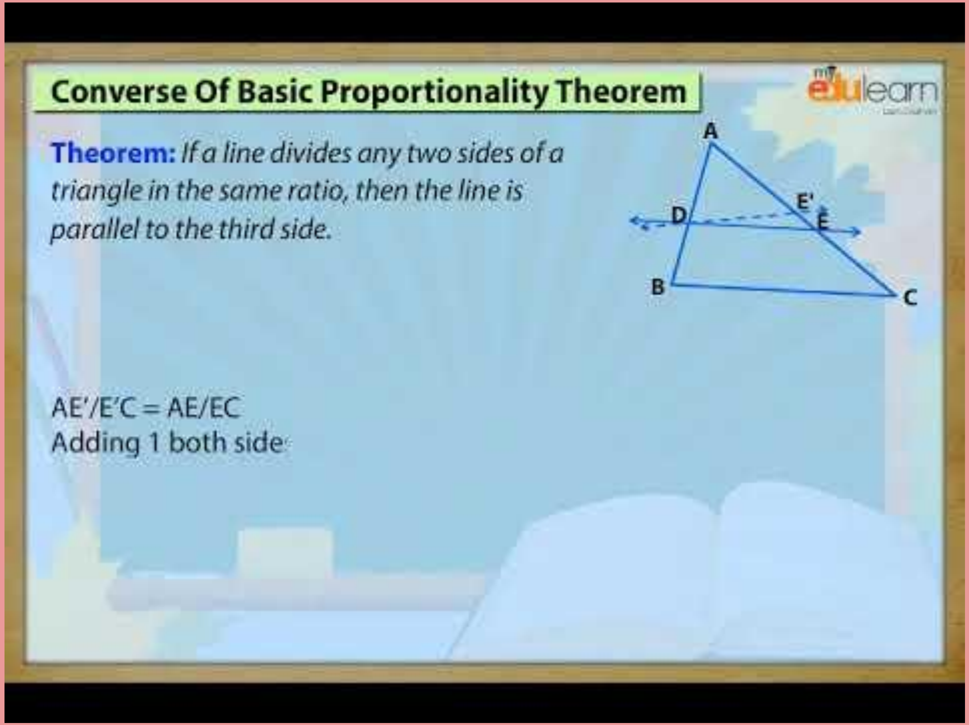
Converse of the Basic Proportionality Theorem.

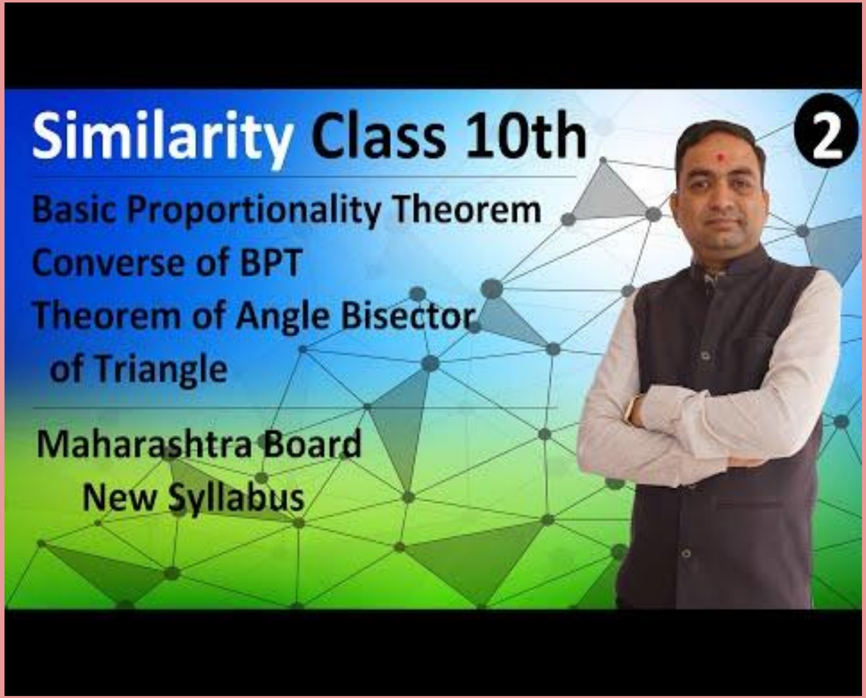
If a line intersects two sides of a triangle , and cuts off segments proportional to these two sides, the it is parallel to the third side.

$$\frac{AB}{AD} = \frac{AC}{AE} \Rightarrow$$



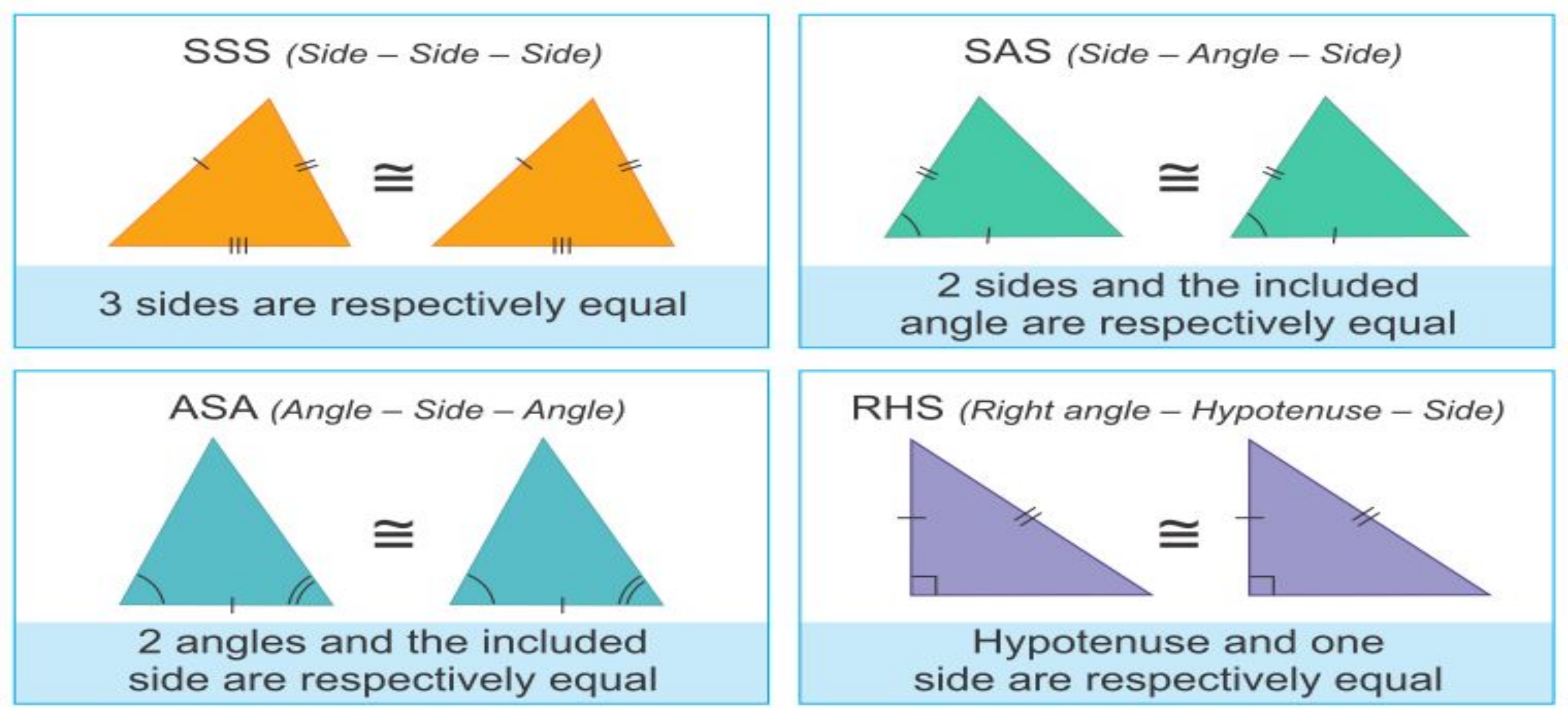
Converse of BPT





Similarity of triangles

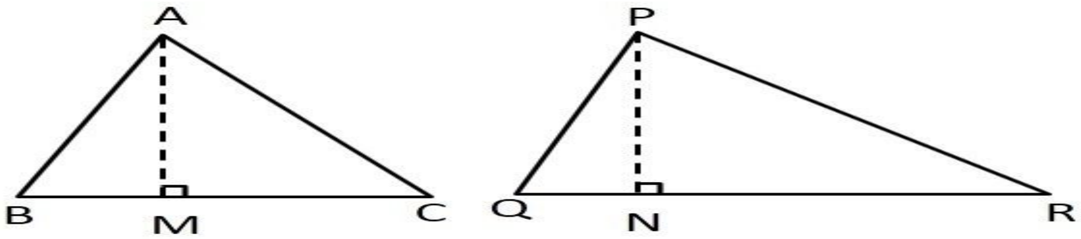
kotak
Education Foundation



Theorem : Areas of similar triangles.

The ratio of the areas of two similar triangles is equal to the square of ratio of their corresponding sides.

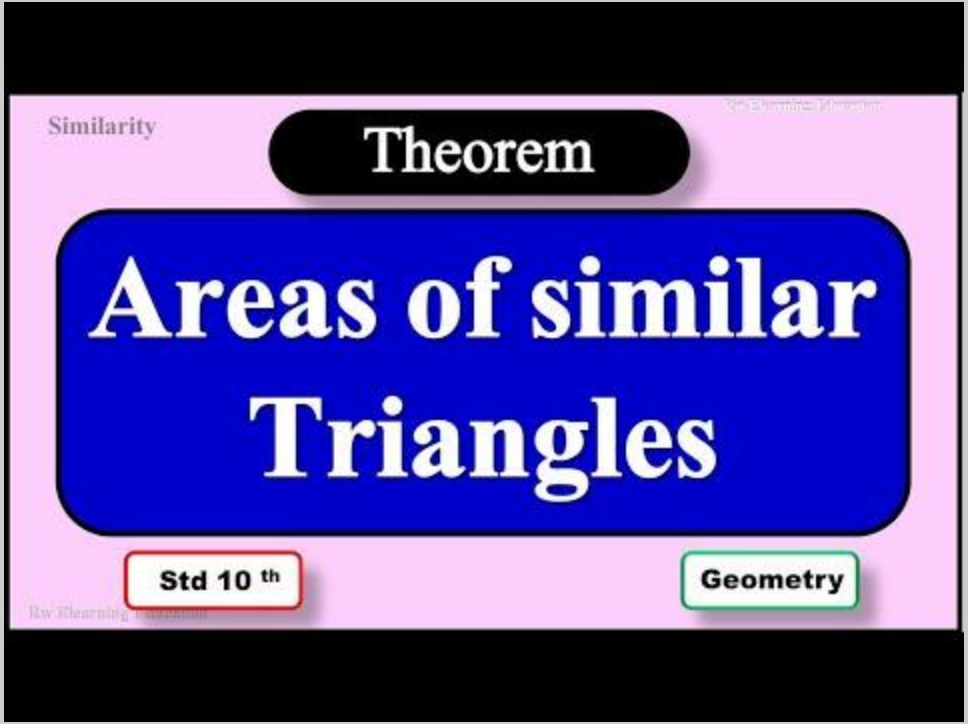
Given: $\Delta ABC \sim \Delta PQR$

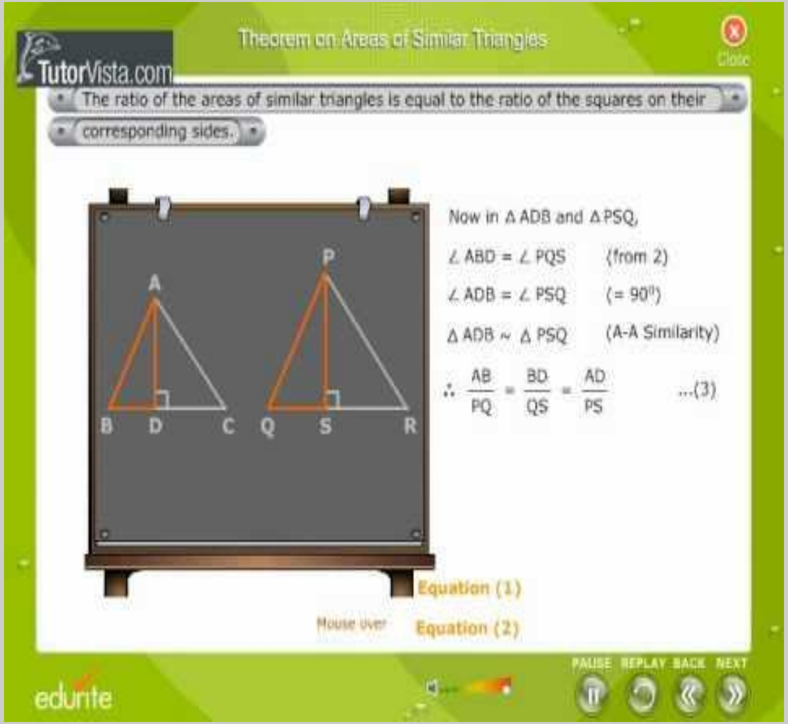


To Prove: $\frac{ar\,(ABC)}{ar\,(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

Construction: Draw $AM \perp BC$ and $PN \perp QR$.

https://www.youtube.com/watch?v=pd331nTd-gE&t=106s

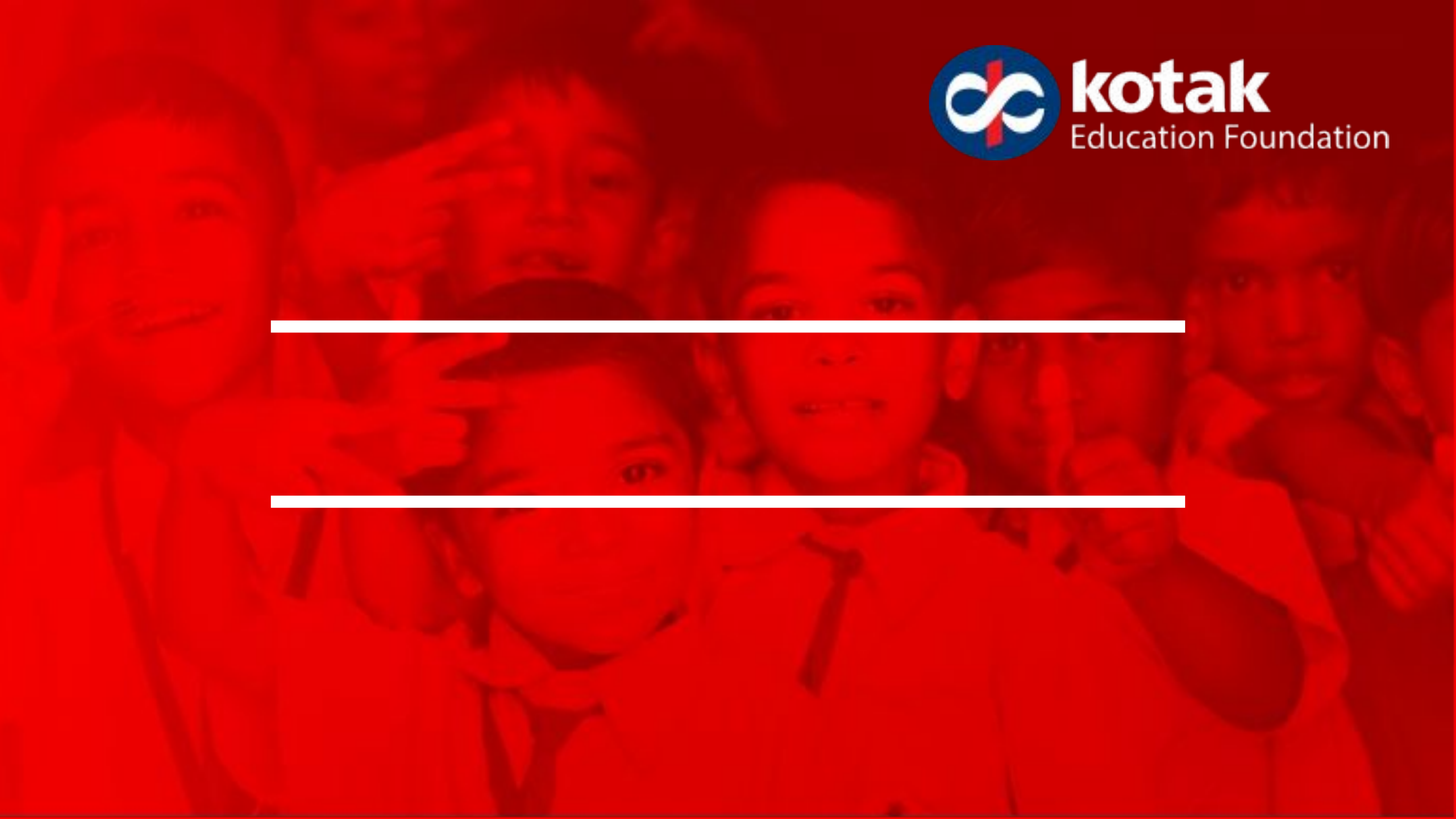




Evaluation

https://docs.google.com/forms/d/15FVfH5sTyMRJICuR2xe_lKDwKfgl8Lw-VzCViQtPXRM/edit


kotak
Education Foundation



kotak
Education Foundation